Appendixes

Theorem 1. *Given a node v, if exists a neighbor node v' of v, the tree node $X(v')$ is a passing node on the path from root node r to v in the graph-decomposed tree Λ rooted by node r.*

Proof [**For Theorem 1**] Considering a node v and its neighbor nodes $N(v)$, node v can be abstracted as a tree node $X(v)$, satisfying $N(v) = X(v) - v$. If exists a tree node $X(v')$, satisfying that $X(v')$ is a parent of $X(v)$, then $v' \in X(v) - v$. v' is connected with all other neighbor nodes $X(v) - v - v'$ after v is deleted. Thus, if exists a tree node $X(v'')$, satisfying that $X(v'')$ is a parent of $X(v')$, such that $X(v'') \in (X(v) - v - v') \cup (X(v') - v')$. Similarly, all tree nodes abstracted from neighbor nodes of v can be found into a path from root node to v.

Lemma 1. *Given a query node q and a data node v, if exists a minimum common ancestor v' of q and v, the shortest path dist(q, v) is equal to the sum of shortest paths dist(q, v') and dist(v', v).*

Proof [**For Lemma 1**] If there exists one other node v'', satisfying the shortest path dist(q, v) is equal to the sum of shortest paths dist(q, v'') and dist(v'', v), then there must exist one or more across-path edges to connect the node v''. However, it is impossible to exist one across-path edge in graph-decomposed tree according to Theorem 1, because any node v and its neighbor nodes only be found into the path from root node r to v.