# Efficient Improvement of Lower Bounds in Equitable Coloring

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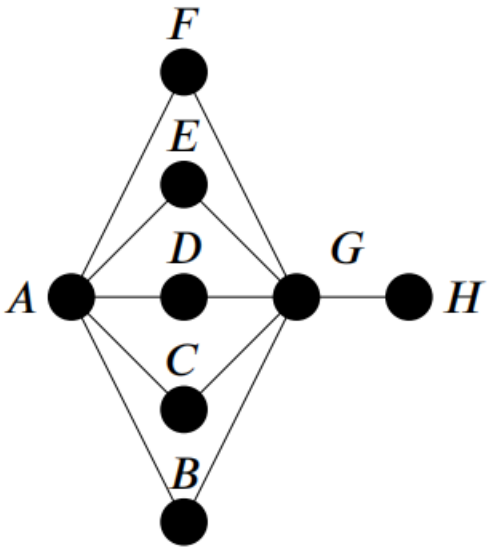
Frontiers of Computer Science, DOI: 10.1007/s11704-025-50183-9

# Problems & Ideas

- Problems of Equitable Graph Coloring Problem:
  - An equitable p-coloring of a graph is a partition of its vertices into $p$ disjoint independent color sets so that the cardinalities of any two sets differ by one at most.
- Ideas: We introduce a new three-stage approach to address this problem and propose three techniques to improve solving efficiency.




A graph with significant differences in color classes caused by traditional coloring methods

# Main Contributions

- Contributions:
  - We introduce a new three-stage approach to address this problem. The approach begins by obtaining an initial solution, then utilizes Integer Linear Programming (ILP) formulations to model and solve ECP. Finally, SAT formulations are employed to enhance the results obtained from ILP solvers.
  - Our approach improves the lower bound of the equitable chromatic number.

| Instance | start | M2P | | ce-ff | | ce-uf | | ce-uf-fm | | Note |
|---|---|---|---|---|---|---|---|---|---|---|
| id | lb | lb | time(s) | lb | time(s) | lb | time(s) | lb | time(s) | |
| 1-Insertions_6.col | 2 | 5 | 586.84 | 4 | 2.87 | 5 | **240.55** | 5 | 460.48 | |
| 2-Insertions_5.col | 2 | 4 | 13.67 | 4 | **12.84** | 4 | 13.10 | 4 | 87.18 | |
| 3-Insertions_5.col | 2 | **4** | 721.82 | **4** | 740.82 | **4** | **646.23** | 3 | 0.06 | |
| 4-FullIns_4.col | 6 | **8** | 212.63 | **8** | **54.19** | 7 | 17.40 | **8** | 66.30 | $\chi_{eq} = 8$ |
| 4-FullIns_5.col* | 6 | 6 | TO | 6 | TO | 6 | TO | **7** | 1.76 | |
| ash608GPIA.col | 3 | 4 | **5.66** | 4 | 8.02 | 4 | 34.57 | 4 | 6.10 | $\chi_{eq} = 4$ |
| ash958GPIA.col | 3 | 4 | **18.30** | 4 | 21.77 | 4 | 21.60 | 4 | 19.16 | $\chi_{eq} = 4$ |
| C2000.5.clq | 16 | 16 | TO | 16 | TO | 16 | TO | **17** | 311.94 | |
| C2000.9.clq | 71 | 71 | TO | 71 | TO | 71 | TO | **75** | 1229.39 | |
| DSJC1000.1.col | 6 | 6 | TO | 6 | TO | 6 | TO | 6 | TO | |
| DSJC1000.5.col | 15 | 15 | TO | 15 | TO | 15 | TO | **18** | 854.20 | |
| DSJC1000.9.col* | 64 | 64 | TO | 64 | TO | 64 | TO | **117** | 890.14 | |
| DSJC125.5.col* | 10 | 13 | 743.52 | 13 | 111.65 | 15 | 116.26 | 15 | **36.78** | |
| DSJC125.9.col* | 34 | 34 | TO | 43 | 436.81 | 42 | 442.42 | **44** | 4.98 | $\chi_{eq} = 44$ |
| DSJC250.1.col | 4 | 6 | 94.68 | 6 | 47.81 | 6 | 46.53 | 6 | **34.29** | |
| DSJC250.5.col* | 12 | 12 | TO | 13 | 1222.56 | **16** | 765.89 | **16** | **64.66** | |

Comparison of some experimental results