Massively Parallel Algorithms for Fully Dynamic All-Pairs Shortest Paths

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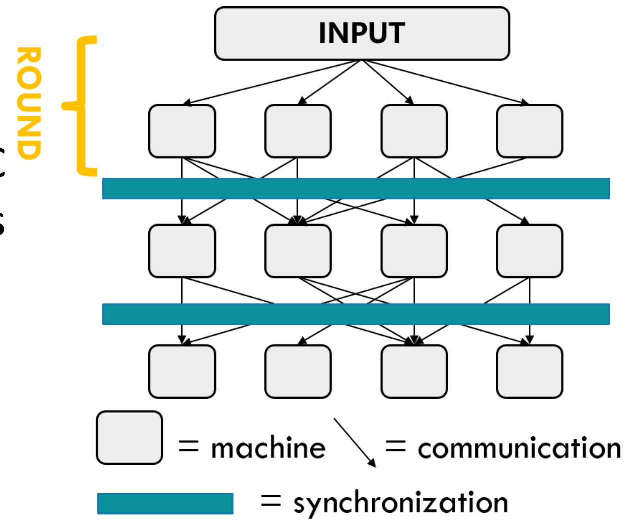
Problems & Ideas

- Problem： Designing fully dynamic APSP algorithm in the MPC model:
  - Most of parallel APSP algorithms in the MPC model are static, no dynamic APSP algorithms exist in the MPC model up to now.
  - A direct implementation of the sequential dynamic APSP algorithms in the MPC model result in a high round complexity.



The computation process of the MPC model

- Ideas: we combine graph techniques and algebraic methods to reduce the round complexity of our parallel fully dynamic APSP algorithm in the MPC model
  - We combine the restricted Bellman-Ford Algorithm with the matrix multiplication on semirings that can update both the shortest distances and paths between nodes efficiently.
  - To reduce the round and memory complexities, we utilize several techniques long-path decomposition, the matrix multiplication on semirings, and the blocked Floyd-Warshall algorithm.

Main Contributions

- Contributions:
  - We propose the first parallel fully dynamic APSP algorithm in the MPC model with $O(n^{\frac{2}{3}-\frac{\alpha}{6}} \log n / \alpha)$ worst-case update rounds, where n is the number of nodes in a graph and $\alpha \in (0,1)$ is a constant;
  - Compared with the existing fastest MPC algorithm, the parallel algorithm proposed in this paper reduces the round complexity by a factor of $O(n^{\frac{1}{3}+\frac{\alpha}{6}} / \log n)$.

Table 1 Comparing Our Parallel Fully Dynamic APSP Algorithm with the Existing Works

	Rounds	Memory	CC	EW	Query	Type
Karczmarz et al. [8]	$O(d), (d \in [1, n])$	$\tilde{O}(n^3)$	$\tilde{O}(n^3)$	Real	Distances	Deterministic
Cao et.al [9]	$O(n^{3/2+o(1)})$	$\tilde{O}(n^3)$	$\tilde{O}(n^3)$	Integer	Distances/Paths	Randomized
AEV of Hajiaghayi et al. [10]	$O(n/\alpha)$	$O(n^{3-\alpha/2})$	$O(n^3 \log n)$	Real	Distances/Paths	Deterministic
ADP of Abraham et al. [4]	$\tilde{O}(n^{2+2/3})$	$O(n^{\alpha})^*$	$\tilde{O}(n^{2+2/3})$	Real	Distances/Paths	Randomized
This work	$O(n^{\frac{2}{3}-\frac{\alpha}{6}} \log n/\alpha)$	$O(n^{3-\alpha/2})$	$O(n^3 \log n)$	Real	Distances/Paths	Randomized

Remark: (1) AEV: an extended version; ADP: a direct parallelization; CC： computation complexity; EW: edge weight
(2) $*$ means that the total memory required is the sum of memory of O(1) machines.