

Mathematical Model and Simulated Annealing Algorithm for Chinese High School Timetabling Problems under the New Curriculum Innovation – Supplementary Material

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SI. EXAMPLES OF COMPULSORY AND OPTIONAL TIMETABLES

Table SI gives an example of teacher assignment for classes, where sc is the code that represents subjects in the Subjects column. For instance, $sc = 1$ represents the Chinese. The Teachers column gives the last name of teachers who teach the corresponding subjects in the Subjects column and the classes in the School classes column. An example structure of Q_c is shown in Table SII, which shows a timetable of four school classes with five timeslots. At timeslot $p = 1$, school class-2-1 is assigned the subject 1 (Chinese) with teacher Zhao according to Table SI. The subject 2 (Math) with teacher Zhou is assigned at $p = 3$, and at $p = 4$ and 5, subject 3 (English) and subject 1 (Chinese) with teacher Zhen and Zhao are assigned, respectively. Slots that are filled with 0 represent idle timeslots.

Table SIII shows an example list of optional classes and teacher assignments. The Optional classes column gives the name of optional classes and o is the corresponding code for optional classes. For example, $o = 1$ represents the first class of Physics. The last names of the assigned teachers for optional classes are given in the Teachers column. As an example of Q_o , Table SIV shows a timetable for seven optional classes with five timeslots. In Table SIV, Physics-1 is assigned to timeslots $p = 1$ and 5 with teacher Feng, Physics-2 is assigned to $p = 1$ and 2 with the same teacher, and the assignments for other optional classes can be easily concluded from Table SIV.

Hard and soft constraints can be easily evaluated using Tables SII and SIV. For example, in Table SII, teacher Zhao has a collision at $p = 1$ and $p = 5$, which breaks the hard constraint H3, and in Table SIV, teacher Liu of Physics-3 has idle timeslots between two time assignments within one workday, which violates constraint S1. Constraint H4 is one of the most critical constraints in CHSTPs, where the selection-schemas of students may cause severe violations. To evaluate H4, Q_c and Q_o should be considered corporately. For example, if a student in school class-2-1 is grouped into a selection-schema (Physics-1, Chemistry-2, History-2), then constraint H4 is violated at $p = 1, 4$, and 5 according to Tables SIII and SIV. At $p = 1$, the time assignments for the Chinese and Physics-1 collides; at $p = 4$ the time assignments for the History-2 and English collides; and the time assignments for the Chinese, Chemistry-2, and History-2 collides at $p = 5$, which results in two counts for violation of H4.

TABLE SI
TEACHER ASSIGNMENTS FOR CLASSES

sc	Subjects	Teachers	School classes
1	Chinese	Zhao	2-1, 2-2
		Qian	2-3, 2-4
2	Math	Zhou	2-1, 2-2
		Wu	2-3, 2-4
3	English	Zhen	2-1, 2-2
		Wang	2-3, 2-4

TABLE SII
STRUCTURE OF SCHOOL CLASS TIMETABLE

School classes (c)	Timeslots				
	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
2-1	1	0	2	3	1
2-2	1	2	3	0	1
2-3	3	2	3	1	0
2-4	0	3	0	2	1

TABLE SIII
EXAMPLE LIST OF OPTIONAL CLASSES AND TEACHER ASSIGNMENTS

Optional classes	o	Teachers
Physics-1	1	Feng
Physics-2	2	Feng
Physics-3	3	Liu
Chemistry-1	4	Han
Chemistry-2	5	Han
History-1	6	Li
History-2	7	Li

TABLE SIV
STRUCTURE OF OPTIONAL CLASS TIMETABLE

Optional classes(o)	Timeslots (p)				
	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
1	1	0	0	0	1
2	1	1	0	0	0
3	0	0	1	0	1
4	1	1	0	0	0
5	0	0	0	0	1
6	1	0	1	0	0
7	0	0	0	1	1

SII. DATASET CHARACTERIZATION

In Table SV, for the instances belonging to SM-1, the number of selection-schemas in a school class is randomly determined in the range $[1, 5]$, and the range is set to $[5, 10]$ and $[10, 15]$ for instances belonging to SM-2 and SM-3, respectively. Therefore, instances in different selection-schema groups have different levels of difficulty with regard to student constraints. n_c denotes the number of school classes, n_t is the total number of teachers, n_m is the number of selection-schemas in all school

classes, and n_o is the number of optional classes. The total number of timeslots in one week is set to $n_p=35$, namely, five days with seven timeslots per day. Values that belong to different instances in one selection-schema group are separated by “/”. The rules for deciding other parameters in a dataset are given as follows: the maximum number of optional classes for each optional subject is set to half of the number of school classes; namely, $n_c/2$. Therefore, for each instance, the maximum value of n_o is $6 \times n_c/2$. The maximum number of classes assigned to a teacher is set to two. Thus, the number of teachers in an instance is approximately $(3 \times n_c + n_o)/2$.

TABLE SV
SUMMARY OF THE GENERATED TEST SETS

		n_c	n_p	n_s	n_m	n_o
C10	SM-1	10	35	32/31/33/32/28	38/26/31/30/18	29/28/30/29/22
	SM-2	10		33/33/33/33/33	70/72/83/72/82	30/30/30/30/30
	SM-3	10		33/33/33/33/33	114/118/117/117/119	30/30/30/30/30
C30	SM-1	30		90/90/92/91/92	87/89/92/108/95	87/85/89/88/89
	SM-2	30		93/93/93/92/93	232/240/226/229/239	90/90/90/89/90
	SM-3	30		93/93/93/93/93	372/368/369/378/352	90/90/90/90/90
C50	SM-1	50		150/147/149/147/146	168/146/148/148/132	145/143/146/140/139
	SM-2	50		153/153/153/153/153	386/357/375/386/380	150/150/150/150/150
	SM-3	50		153/153/153/153/153	615/622/631/629/625	150/150/150/150/150

TABLE SVI
INITIAL COSTS FOR THE GENERATED TEST SETS

		(Obj_h, Obj_s)
C10	SM-1	(240, 324)/(161, 303)/(208, 321)/(200, 312)/(116, 287)
	SM-2	(442, 323)/(443, 316)/(500, 323)/(450, 304)/(487, 315)
	SM-3	(696, 315)/(734, 313)/(724, 310)/(720, 320)/(721, 332)
C30	SM-1	(575, 955)/(583, 917)/(596, 963)/(712, 939)/(596, 935)
	SM-2	(1408, 959)/(1460, 982)/(1357, 986)/(1417, 981)/(1459, 996)
	SM-3	(2230, 985)/(2171, 973)/(2260, 1008)/(2324, 1005)/(2086, 990)
C50	SM-1	(1054, 1613)/(922, 1600)/(980, 1621)/(977, 1574)/(858, 1592)
	SM-2	(2339, 1614)/(2245, 1641)/(2255, 1640)/(2444, 1628)/(2291, 1629)
	SM-3	(3695, 1628)/(3813, 1621)/(3713, 1614)/(3727, 1578)/(3730, 1604)

To have an intuitive sense of the difficulty of the generated datasets, the initial costs, which can reflect the complexity of instances to some extent, of each instance in the generated datasets are given in Table SVI. From Table SVI we can see that the initial soft costs of instances in the same dataset are approximately equal, while the initial hard costs vary a lot among different selection-schema groups. For example, in dataset C10 the average initial hard costs are 185, 464, and 719 for SM-1, SM-2, and SM-3, respectively, where the average initial hard costs of SM-3 are approximately twice that of SM-2, and four times that of SM-1. The same phenomenon can be observed in C30 and C50. However, the average initial soft costs for SM-1, SM-2, and SM-3 are 309, 316, and 318, respectively, which are very close. Therefore, different settings for the number of selection-schemas for school classes can result in different difficulties in regard to the hard constraints of instances. Moreover, by comparing the initial costs of instances in the same selection-schema group in different datasets, it can be seen that there are distinct differences in both hard and soft costs. For example, for SM-1 the average initial hard costs of instances in C10, C30, and C50 are 185, 612, and 958, respectively, and soft costs are 309, 942, and 1600, respectively. Therefore, different settings for the number of school classes can result in different difficulties in regard to both hard and soft constraints of instances.

To better understand the generated datasets, the initial costs listed in Table SVI are plotted in Fig.S1. From Fig.S1 (a) we

can see that the difference of initial feasibility values among different selection-schema groups within the same datasets are distinct; and in Fig.S1 (b), the initial objective values within the same dataset are almost the same but vary a lot among different datasets. Therefore, the range of the number of selection-schemas in one school class is a vital factor that affects the difficulty levels in finding feasible solutions, while the number of school and optional classes, teachers, and the total number of selection-schemas mainly affect the difficulty levels in finding the optimal timetables.

The principle of generating simulative datasets is to cover various scales of HSTPs that are probably faced by different high schools. As can be seen from Table SV in the Supplementary Material, the number of school classes varies from 10 to 50, the number of teachers and optional classes vary from 30 to 150, and the total number of selection schema ranges from 30 to 600, which means that the generated instances are representative for high schools that are in small, middle and large-scales. Furthermore, Table SVI lists the initial hard and soft costs of the generated instances. The initial hard costs can be used to evaluate the difficulty levels in finding feasible timetables while the initial soft costs in finding the optima. As can be seen from Table SVI, the initial hard costs vary from 100 to 3800, and initial soft costs range from 300 to 1600, which provides adequate scales of difficulty levels for both finding feasible and optimal solutions. Therefore, the generated datasets are capable of simulating the realistic cases in high schools with different scales and for illustrating the characteristics of the proposed model.

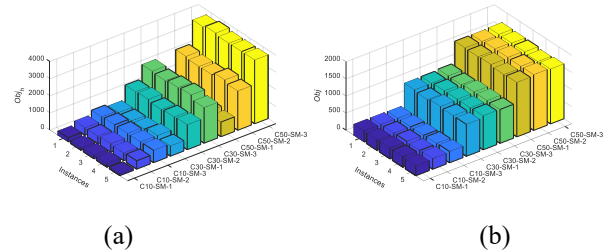


Fig.S1. Initial costs of the generated datasets. (a) Hard costs. (b) Soft costs.

III. EXPERIMENTS ON REAL DATASETS

The real instances are collected from high schools in South China. The collected data are desensitized and named with “SchoolX”, where X represents Arabic numerals 1-10. This section at first describes the properties of the real instances, then follows the experimental results.

A. The properties of the real data

Table SVII presents the properties of ten real instances. The name in the first row from left to right represents the number of school classes, the number of optional classes, the number of teachers, the number of subjects, the number of total selection-schemas in all school classes, the number of days in one week, the number of timeslots in one week, density of the compulsory timetable Q_c , density of the optional timetable Q_o , respectively.

TABLE SIX (CONTINUE)

		Subjects																
		1	2	3	4	5	6	7	8	9	11	12	13	14	20	21	22	23
oc-2	Teacher	--	--	--	3862	--	--	--	--	--	--	--	--	--	--	--	--	--
	Du-1	--	--	--	3	--	--	--	--	--	--	--	--	--	--	--	--	--
	Du-2	--	--	--	0	--	--	--	--	--	--	--	--	--	--	--	--	--
oc-3	Teacher	--	--	--	--	3865	--	--	--	--	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	3	--	--	--	--	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	0	--	--	--	--	--	--	--	--	--	--	--	--
oc-4	Teacher	--	--	--	--	3864	--	--	--	--	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	3	--	--	--	--	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	0	--	--	--	--	--	--	--	--	--	--	--	--
oc-5	Teacher	--	--	--	--	--	3867	--	--	--	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	2	--	--	--	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	0	--	--	--	--	--	--	--	--	--	--	--
oc-6	Teacher	--	--	--	--	--	3868	--	--	--	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	2	--	--	--	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	0	--	--	--	--	--	--	--	--	--	--	--
oc-7	Teacher	--	--	--	--	--	3867	--	--	--	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	2	--	--	--	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	0	--	--	--	--	--	--	--	--	--	--	--
oc-8	Teacher	--	--	--	--	--	3868	--	--	--	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	2	--	--	--	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	0	--	--	--	--	--	--	--	--	--	--	--
oc-9	Teacher	--	--	--	--	--	--	3870	--	--	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	--	2	--	--	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	--	0	--	--	--	--	--	--	--	--	--	--
oc-10	Teacher	--	--	--	--	--	--	3871	--	--	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	--	2	--	--	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	--	0	--	--	--	--	--	--	--	--	--	--
oc-11	Teacher	--	--	--	--	--	--	--	3873	--	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	--	--	2	--	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	--	--	0	--	--	--	--	--	--	--	--	--
oc-12	Teacher	--	--	--	--	--	--	--	3872	--	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	--	--	2	--	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	--	--	0	--	--	--	--	--	--	--	--	--
oc-13	Teacher	--	--	--	--	--	--	--	3872	--	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	--	--	2	--	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	--	--	0	--	--	--	--	--	--	--	--	--
oc-14	Teacher	--	--	--	--	--	--	--	--	3875	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	--	--	--	2	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	--	--	--	0	--	--	--	--	--	--	--	--
oc-15	Teacher	--	--	--	--	--	--	--	--	3874	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	--	--	--	2	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	--	--	--	0	--	--	--	--	--	--	--	--
oc-16	Teacher	--	--	--	--	--	--	--	--	3875	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	--	--	--	2	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	--	--	--	0	--	--	--	--	--	--	--	--
oc-17	Teacher	--	--	--	--	--	--	--	--	3874	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	--	--	--	2	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	--	--	--	0	--	--	--	--	--	--	--	--
oc-18	Teacher	--	--	--	--	--	--	--	--	3876	--	--	--	--	--	--	--	--
	Du-1	--	--	--	--	--	--	--	--	2	--	--	--	--	--	--	--	--
	Du-2	--	--	--	--	--	--	--	--	0	--	--	--	--	--	--	--	--

TABLE SX
SELECTION SCHEMAS IN SCHOOL6

Selection schemas	Contained classes			Selection schemas	Contained classes			Selection schemas	Contained classes		
ss-1	sc-1	--	--	ss-11	sc-4	oc-3	oc-7	ss-21	sc-6	oc-12	oc-17
ss-2	sc-2	oc-4	oc-6	ss-12	sc-4	oc-3	oc-8	ss-22	sc-7	oc-2	oc-14
ss-3	sc-2	oc-4	oc-5	ss-13	sc-5	oc-6	oc-10	ss-23	sc-7	oc-2	oc-15
ss-4	sc-2	oc-3	oc-7	ss-14	sc-5	oc-5	oc-10	ss-24	sc-7	oc-1	oc-17
ss-5	sc-3	oc-7	oc-14	ss-15	sc-5	oc-9	oc-8	ss-25	sc-7	oc-1	oc-18
ss-6	sc-3	oc-7	oc-15	ss-16	sc-6	oc-13	oc-14	ss-26	sc-8	oc-13	oc-14
ss-7	sc-3	oc-6	oc-16	ss-17	sc-6	oc-13	oc-15	ss-27	sc-8	oc-13	oc-15
ss-8	sc-3	oc-5	oc-16	ss-18	sc-6	oc-11	oc-16	ss-28	sc-8	oc-11	oc-18
ss-9	sc-4	oc-4	oc-6	ss-19	sc-6	oc-12	oc-13	ss-29	sc-8	oc-12	oc-18
ss-10	sc-4	oc-4	oc-5	ss-20	sc-6	oc-11	oc-17				

TABLE SXI
RESULTS ON REAL DATA

	School1	School2	School3	School4	School5	School6	School7	School8	School9	School10
c_h^*	0	0	0	0	0	0	0	0	0	0
\bar{c}_h	0	0.3	2.2	0	0	0	0.8	0	5.7	0
$Best_h\%$	100	100	100	100	100	100	100	100	100	100
c_s^*	646	254	130	160	375	179	328	101	380	162
\bar{c}_s	650	271	151	175	377	182.8	336	104	425	164
$Best_s\%$	69.8	64.1	63.7	70.7	69.0	57.2	66.2	83.5	44.4	74.7
\overline{CPU}	1161	135	49	85	412	63	299	115	783	132

B. Experimental results on real data

Table SXI shows the experimental results of 10 real schools with the parameters in Table II. Table SXII is the results on School2, School3, School7, and School9 with $L_1=500$ and other parameters unchanged.

TABLE SXII
RESULTS ON SCHOOL2, SCHOOL3, SCHOOL7, AND SCHOOL9 WITH $L_1=500$

	School2	School3	School7	School9
c_h^*	0	0	0	0
\bar{c}_h	0	0	0.1	0
$Best_h\%$	100	100	100	100
c_s^*	267	134	329	396
\bar{c}_s	274	154	226	412
$Best_s\%$	61.7	62.8	66.6	42.4
\overline{CPU}	138	58	382	932

SIV. CORRESPONDING TABLES AND FIGURES

TABLE SXIII
THREE EXAMPLES FOR SELECTION-SCHEMAS IN SCHOOL CLASS-2-1

Selection-schema 1	Physics-1	Chemistry-1	Biology-1
Selection-schema 2	History-1	Chemistry-1	Biology-1
Selection-schema 3	Physics-2	Chemistry-1	Biology-1

TABLE SXIV
TWO TIMETABLES FOR AN ENGLISH TEACHER
MO-FR: MONDAY-FRIDAY.

SC: SCHOOL CLASS					
	Mo	Tu	We	Th	Fr
Timetable-1	SC-2-1	SC-2-1	SC-2-1		
			SC-2-2	SC-2-2	SC-2-2
Timetable-2	SC-2-1	SC-2-1	SC-2-1		
	SC-2-2	SC-2-2	SC-2-2		

TABLE SXV
DETAILED VIOLATIONS OF HARD COST IN SM-2 AND SM-3 OF C10

	SM-2					SM-3				
	1	2	3	4	5	1	2	3	4	5
$r_{hs}(\%)$	87.8	93.6	94.0	90.0	94.8	91.4	93.3	94.9	94.0	94.1
$r_{ht}(\%)$	12.1	6.3	5.9	9.9	5.1	8.5	6.6	5.0	5.9	5.8
$r_s(\%)$	46.5	57.2	67.2	64.5	68.9	7.6	81.5	76.7	79.6	79.4
$r_t(\%)$	14.5	12.7	14.8	19.0	14.2	32.4	30.3	23.0	26.6	26.9