Normalized Fourier induced coupled PINNs to solve the Dirichlet biharmonic equations in large scale domain

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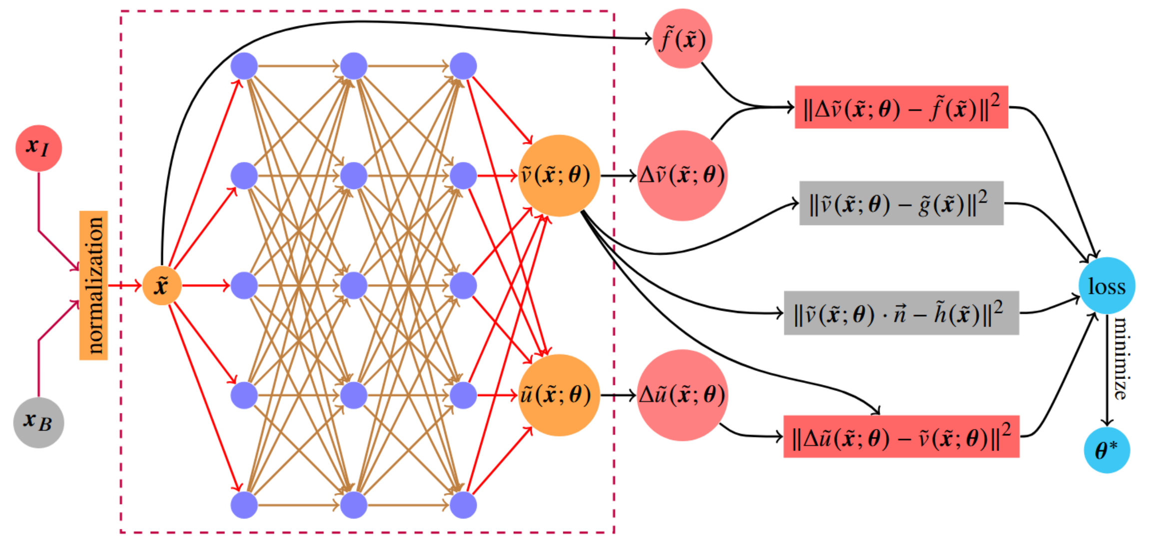
Frontiers of Computer Science, DOI: 10.1007/s11704-025-50366-4

Problems & Ideas

- Problems of PINNs for solving large-scale biharmonic equations:
  - Wide-range inputs cause the network to dilute important data features.
  - Data dilution directly leads to slow convergence and reduced model accuracy.
- Ideas: By integrating the normalizing technique with a Fourier-induced mapping, the data dilution will be reduced, ensuring both accuracy and efficiency when solving large-scale problems.

$$\begin{cases} \Delta^2 u(\boldsymbol{x}) = \sum_{i=1}^{d} \frac{\partial^4 u}{\partial x_i^4} + \sum_{i=1}^{d} \sum_{j=1, j\neq i}^{d} \frac{\partial^4 u}{\partial x_i^2 \partial x_j^2} = f(\boldsymbol{x}) \ \text{ in } \Omega \\ u(\boldsymbol{x}) = g(\boldsymbol{x}) \ \text{ and } \ \frac{\partial u(\boldsymbol{x})}{\partial \vec{n}} = h(\boldsymbol{x}) \ \text{ on } \partial\Omega \end{cases}$$

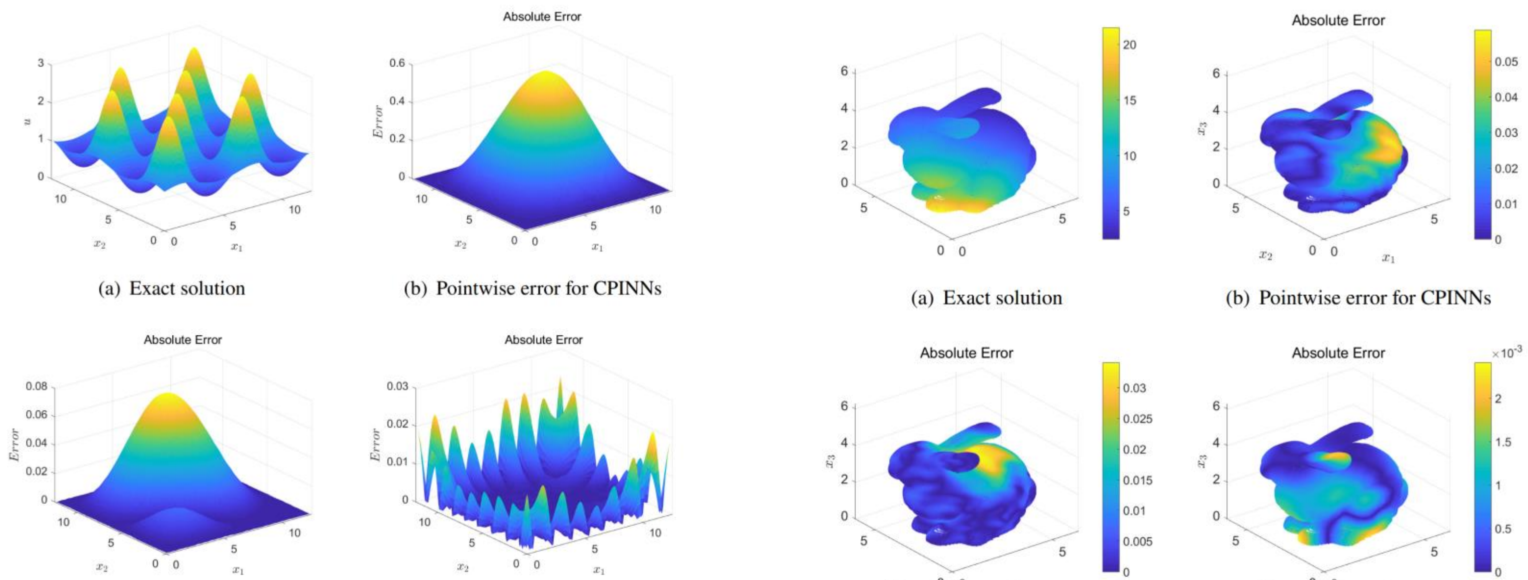
The formulation of biharmonic equations with Dirichlet boundary conditions



Schematic of the proposed NFCPINNs architecture. The input data $\boldsymbol{x}$ is normalized into a unit space ˜$\boldsymbol{x}$, then obtain the solution **u** and intervale variable **v** by a Fourier DNN, with the total loss subsequently formed based on the correspondingly normalized physical constraints.

Main Contributions

- Contributions:
  - The normalization technique transforms the physical domain into a unit space, mitigating the gradient dissipation and feature dilution issues under large-scale inputs for solving biharmonic equations.
  - Fourier-induced DNN is configured as the solver for NCPINNs, mimicking Fourier expansion to enhance nonlinearity of DNN.
  - Numerical experiments demonstrate the proposed NFCPINNs outperforms existing approaches in terms of accuracy, convergence speed when tackling large-scale problems.



Relative and absolute error for different methods, Left: 2-D rectangle domain problem Right:3-D irregular domain problem