On the exact quantum query complexity of MOD and EXACT functions

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Problems & Ideas

- Problems: The exact quantum query complexity of MOD and EXACT functions are currently not fully characterized.
- Ideas: By observing the construction of functions, we design the optimal quantum algorithms to compute MOD function and some instances of EXACT function.

$$\mathrm{MOD}_m^n(x) = |x| \bmod m$$

$$\mathrm{EXACT}_{k,l}^n(x) = \begin{cases} 1, & \text{if } |x| \in \{k, l\}, \\ 0, & \text{otherwise.} \end{cases}$$

Definitions of MOD and EXACT functions.

Main Contributions

- Contributions:
  - A tight characterization of the exact quantum query complexity of MOD function.
  - A demonstration of non-evasive symmetric functions in the quantum model.
  - A tight characterization of the exact quantum query complexity of EXACT function in some specific scenarios.

Function	Exact quantum query complexity
MOD_m^n	$\lceil n(1-\frac{1}{m})\rceil$
$\mathrm{EXACT}_{k,l}^n$	$\begin{cases}\max\{n-k,l\}-1, & \text{if } l-k\geq 4, k=0,\\ \max\{n-k,l\}-1, & \text{if } l-k\geq 4, k=1, l=n-1.\end{cases}$

Exact quantum query complexity of MOD and EXACT functions.