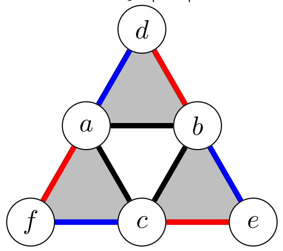
Analysis of Branchings on Claw, Tent and Net in the Proof of Theorem

To destroy each claw, we simply branch on deleting any of its three edges. This results in a branching root 3.

Now we discuss the branching on a tent. The vertices of the tent are labeled as in the following figure. We apply the following branching cases. The edge subset $E' \subseteq E$ given in each of the following cases stands for a branching where exactly the edges in E' are removed. Note that in such a branching, the parameter k is decreased by $|E'|$.



1. $\{(a,d),(b,d)\}$
2. $\{(a,d),(a,b)\}$
3. $\{(b,d),(a,b)\}$
4. $\{(b,e),(c,e)\}$
5. $\{(b,e),(b,c)\}$
6. $\{(b,c),(c,e)\}$
7. $\{(c,f),(a,f)\}$
8. $\{(c,f),(a,c)\}$
9. $\{(a,c),(a,f)\}$

The nine branching cases remove edges in each of the three gray-filled triangles, each branching removes two edges in one of the triangles and each triangle has $\binom{3}{2}$ combinations to consider. By an ad-hoc analysis, one can check that the above branching cases cover all minimal ways of destroying the forbidden structures within the six vertices in the tent. Due to symmetry, we provide below only the detailed analysis for the case where either one edge, say (a,b), in the triangle formed by $\{a,b,c\}$ is removed in a solution, or none of the edges in the triangle is removed but some edge not in the triangle is removed.

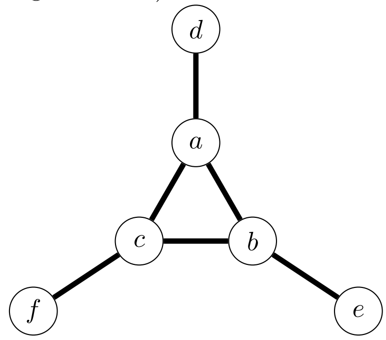
Assume that in a solution the edge (a,b) is removed. After removing this edge, $\{a,d,b,c\}$ forms an induced C_4, and so one of the edges in the cycle must be removed. Removing one of (a,d) and (b,d) has been covered by the second and third branching cases. So, let us assume that in the solution these two edges are not removed.

We continue the analysis. If the edge (b,c) is removed, then after removing (a,b) and (b,c), $\{a,d,b,e,c\}$ forms an induced C_5 and hence at least one edge in the cycle must be removed. Similar to the above analysis, we can assume that none of the edges in the set $\{(a,d),(b,d),(b,e),(c,e)\}$ is removed, since otherwise the solution survives also in some of the above branching cases. So, it only leaves to remove (a,c). After removing (a,c), $\{a,d,b,e,c,f\}$ forms an induced C_6, and one of the edges in the cycle must be removed. Then, it is easy to observe that removing any edge in the cycle further is covered by one of the above branching cases. For instance, if the edge (c,f) is further removed, then it is covered by the branching case 8.

Now we analyze the case where none of the edges in the middle triangle is removed. Then, one edge not in the triangle must be removed. Without loss of generality, let us assume that the edge (a,d) is included in a solution. After removing this edge, there is an induced claw formed by $\{b,d,a,e\}$ with b being the center. One of the two edges (d,b) and (b,e) must be removed. Removing further (d,b) is covered by the first branching case. We consider the removal of (b,e). After removing this edge, $\{b,c,e,f\}$ forms another induced claw with c being the center. One of the edges (c,f) and (c,e) must be further removed. Removing further the edge (c,e) has been covered by the fourth branching case. Removing further the edge (c,f) obtains an induced net, implying that one of the red edges needs to be further removed. However, removing any of the red edges has been covered by one of the above branching cases. This completes the proof that the above 9 branching cases cover all possibilities of destroying all forbidden structures within the tent.

The branching root of the above cases is the positive root of the polynomial $x^2 - 9$ which is bounded by 3.

Now we provide the branching cases for the net (see the figure below).



We have the following branching cases

1. $\{(a,d)\}$
2. $\{(b,e)\}$
3. $\{(c,f)\}$
4. $\{(a,b),(b,c)\}$
5. $\{(a,b),(a,c)\}$
6. $\{(b,c),(a,c)\}$

We claim that the above branching cases cover all possibilities. To this end, observe that if none of the first three cases occurs, then one of the edges in the triangle in the net must be removed. However, removing any of them results in an induced claw which triggers further removing of edges among a, b, and c, which are completely covered by the last three branching cases.

By solving $x^2 - 3x - 3 = 0$, we obtain a branching root at most 3.792 for the above branching cases.