Fixed-Parameter Tractability of Capacitated k-Facility Location

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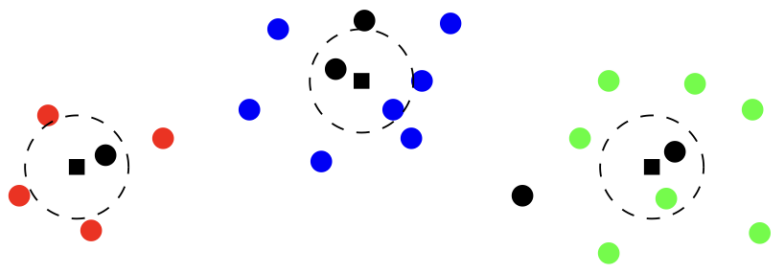
Problems & Ideas

- Problems of approximating capacitated k-facility location:
  - Existing polynomial-time algorithms for the problem cannot yield feasible O(1)-approximation solution.
  - The problem is W[2]-hard if parameterized by k, implying that exactly solving it in FPT(k) time is unlikely.
- Ideas: A two-step sampling-based algorithm for solving the capacitated k-facility location problem, which yields a constant-factor approximation solution in FPT(k) time.

Left: The partition of the client set in a facility location-type instance not involving capacitated constraints; Right: The partition of the client set in an instance of the capacitated k-facility location problem, where the opened facility corresponding to the red clients has a capacity of 7.

Main Contributions

- Contributions:
  - It is proved that a set of randomly sampled clients contains some nearby clients of the opened facilities in an optimal solution. Such a client set is quite valuable in identifying the opened facilities in the approximation solution;
  - A set of facilities with appropriate opening costs and capacities can be selected based on the set of the randomly sampled clients, and the approximation solution can be constructed by connecting each client to a selected facility.



A set of O(k) clients containing some nearby clients of the facilities opened in an optimal solution.

Theorem: Given a real number $\varepsilon>0$, there exists a **(3+ε)**-approximation algorithm for capacitated k-facility location, which runs in **FPT(k)** time.