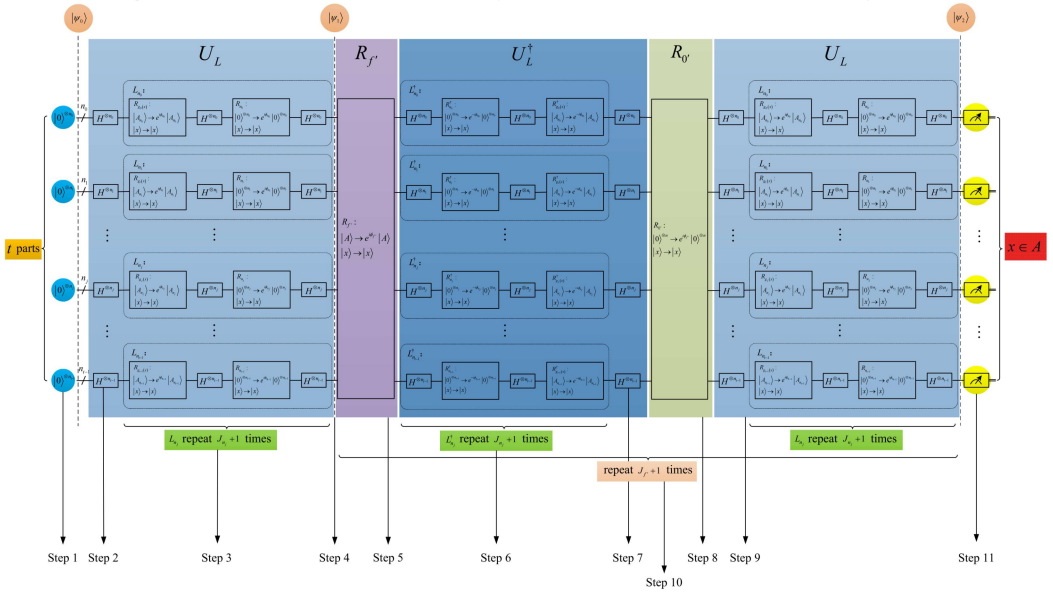
Distributed Exact Generalized Grover's Algorithm

Xu ZHOU*, Xu-Sheng XU, Sheng-Gen ZHENG, Le LUO

Frontiers of Computer Science, DOI: FCS-250239

Problems & Ideas

- Problems of the generalized search within an unordered database:
  - In the NISQ era, quantum computers have a limited number of qubits and are subject to noise interference.
  - Grover's algorithm cannot achieve exact generalized searches and requires multiple applications of multi-qubit gates.
- Ideas: A quantum algorithm that integrates distributed computing with quantum computing, aiming to enhance search precision while considering the feasibility of NISQ era quantum computers.



Quantum circuit of the distributed exact generalized Grover's algorithm with $2 \le t \le n$ computing nodes. By applying the modified Grover's algorithm and quantum exact amplitude amplification algorithm, DEGGA can achieve exact generalized search.

Main Contributions

- Contributions:
  - A Distributed Exact Generalized Grover's Algorithm (DEGGA) is proposed;
  - DEGGA guarantees 100% targets identification;
  - The circuit depth mainly depends on the subfunctions partitioning strategy;
  - The maximum number of qubits required at one node is $\max\{n_0, n_1, \cdots, n_{t-1}\}$, eliminating the need for auxiliary qubits, where $\sum_{j=0}^{t-1} n_j = n$;
  - Exact searches for specific examples (000000 and 111111) are successfully achieved through MindSpore Quantum;
  - Consider the decomposition of multi-qubit gates required in DEGGA.

Table 1 A simple comparison between DEGGA and the existing distributed Grover's algorithms.

	The algorithm in [34]	The algorithm in [35]	The algorithm in [36]	**DEGGA**
1. The number of computing nodes.	2	2^k, where $1 \le k \le n-1$	$\lfloor n/2 \rfloor$	$\mathbf{2 \le t \le n}$
2. Does it solve the generalized search problem?	No	Yes	No	**Yes**
3. Does it solve exactly?	No	No	Yes	**Yes**
4. The maximum number of qubits at a single node.	$n-1$	$n-k$	3	$\mathbf{\max(n_0, n_1, \cdots, n_{t-1})}$
5. The total number of qubits.	$2(n-1)$	$2^k(n-k)$	n	$\boldsymbol{n}$
6. Does it require auxiliary qubits?	No	Yes	No	**No**
7. Does it implement through quantum simulation software?	Yes	No	Yes	**Yes**

Table 2 A simple statistic for the modified Grover's algorithm and DEGGA.

	Modified Grover's algorithm	DEGGA-2	Optimized DEGGA-2	DEGGA-3	Optimized DEGGA-3
1. The number of computing nodes.	1	2	2	3	3
2. Quantum circuit.	Figure 5	Figure 7	Figure 9	Figure 11	Figure 13
3. The maximum number of qubits at a single node.	6	3	3	2	2
4. The maximum circuit depth at a single node.	37	37	37	37	37
5. The maximum number of quantum gates at a single node.	162	87	83	62	60
6. Random seed value.	42	43	44	45	46
7. The sampling results.	Figure 6	Figure 8	Figure 10	Figure 12	Figure 14
8. Target strings.	{000000, 111111}	{000000, 111111}	{000000, 111111}	{000000, 111111}	{000000, 111111}
9. Does it achieve an exact search?	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**

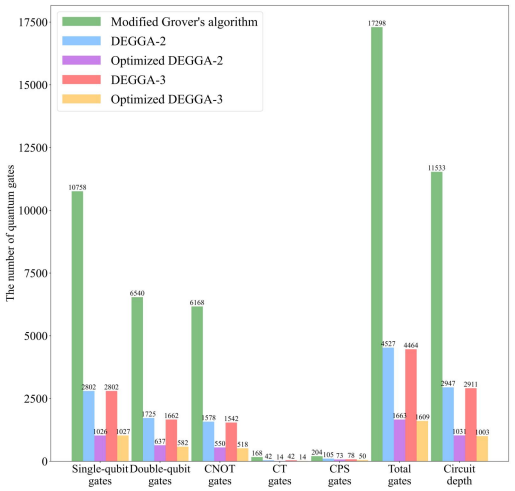


Table 1 compares DEGGA and the existing distributed Grover's algorithms. Table 2 provides statistics for the modified Grover's algorithm and DEGGA across specific examples. The figure on the right compares the modified Grover's algorithm with DEGGA after the decomposition of multi-qubit gates.