Deterministic Streaming Algorithms for Non-monotone Submodular Maximization

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Problems & Ideas

- Problems of **submodular maximization**:

Constrained submodular maximization problem:

$$\max\{ f(S) \mid S \in \mathcal{I} \}$$

$f: 2^N \to \mathbb{R}$: Submodular function (non-monotone)

$\mathcal{I}$: Constraint (d-knapsack, knapsack)

- Ideas for the streaming algorithm framework:
  - **Threshold**: simulating the optimal solution via threshold greedy of density;
  - Lazily running the algorithm for different thresholds when the maximum density is updated;
  - **Repeat**: run the threshold greedy twice, and run a unconstrained submodular maximization algorithm on the first solution.

Proof: by the solution S_1 and S_2 are disjoint

$$f(O) \leq f(O \cup S_1 \cup S_2) + f(O) \leq f(O \cup S_1) + f(O \cup S_2)$$

Main Contributions

- Contributions:
  - We propose a deterministic streaming algorithm framework for non-monotone submodular maximization;
  - We propose a 1-pass streaming algorithm for the d-knapsack constraint has a $1/(4(d+1)) - \varepsilon$ approximation ratio;
  - We propose a 1-pass streaming algorithm with a $1/8 - \epsilon$ approximation ratio to the knapsack constraint;
  - We propose a multi-pass streaming algorithm with $1/6 - \epsilon$ approximation, which stores $O(\tilde{B})$ elements.

Constraint	Ratio	Pass	Monotonicity
d-knapsack	$1/(2d+1) - \epsilon$	1	Monotone
	$1/(4d+4) - \epsilon$	1	Non-monotone
knapsack	$2/5 - \epsilon$	1	Monotone
	$1/2 - \epsilon$	$O(1/\epsilon)$	Monotone
	$1/8 - \epsilon$	1	Non-monotone
	$1/6 - \epsilon$	$O(\log b/\epsilon)$	Non-monotone

Table 1 Deterministic streaming algorithms for non-monotone submodular maximization under d-knapsack constraint and knapsack constraint. The results of this paper are marked in red.