

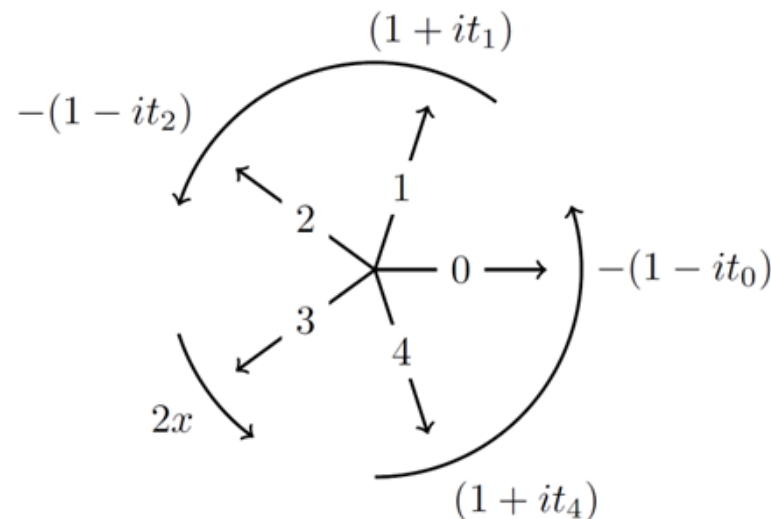
# Revisiting fixed-point quantum search: proof of the quasi- Chebyshev lemma

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# Problems & Ideas

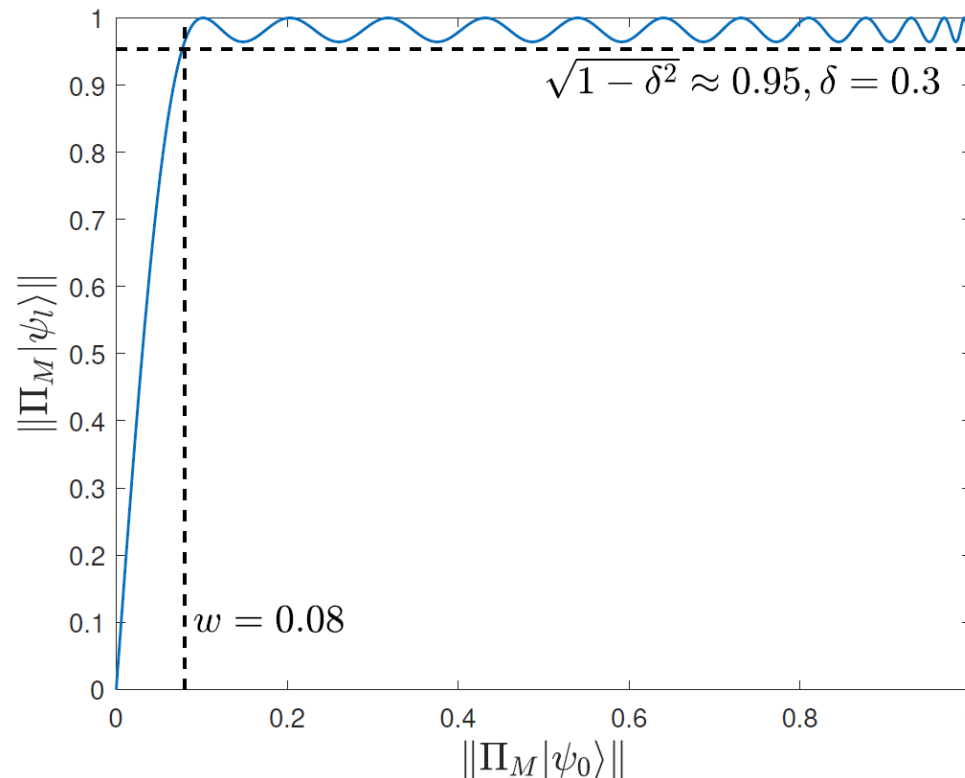
- Problems of fixed-point quantum search [Phys. Rev. Lett. 113, 210501 (2014)]:
  - Its correctness relies on a key lemma regarding the explicit formula of recursive quasi-Chebyshev polynomials (the quasi-Chebyshev lemma), but its proof is not given explicitly.
- Ideas: Combinatorial arguments of recursive quasi-Chebyshev polynomials by counting the weights of specific tilings on  $n$ -stars.



A tiling on the 5-star with positions  $\langle 0,1,2,3,4 \rangle$ . It has one square with weight  $2x$  on position  $\langle 3 \rangle$ , one domino with weight  $-(1-it_2)(1+it_1)$  on positions  $\langle 2,1 \rangle$ , and one domino with weight  $-(1-it_0)(1+it_4)$  on positions  $\langle 0,4 \rangle$ . The weight of this tiling is thus  $2x(1-it_2)(1+it_1)(1-it_0)(1+it_4)$ .

# Main Contributions

- Contributions:
  - The first explicit and detailed proof of the quasi-Chebyshev lemma, thus providing a sound foundation for the correctness of the fixed-point quantum search.



Effect of the fixed-point quantum search: as long as the proportion of marked items is greater than  $w^2$ , the final success probability will be greater than  $1 - \delta^2$ .