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RESEARCH ARTICLE

Dynamical characteristic of measurement uncertainty under Heisenberg spin models with Dzyaloshinskii–Moriya interactions

Ying-Yue Yang[1], **Wen-Yang Sun**[1], **Wei-Nan Shi**[1], **Fei Ming**[1], **Dong Wang**[1,2,†], **Liu Ye**[1]

[1] *School of Physics & Material Science, Anhui University, Hefei 230601, China*
[2] *CAS Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei 230026, China*
Corresponding author. E-mail: † *dwang@ahu.edu.cn*

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The dynamics of measurement's uncertainty via entropy for a one-dimensional Heisenberg XYZ mode is examined in the presence of an inhomogeneous magnetic field and Dzyaloshinskii–Moriya (DM) interaction. It shows that the uncertainty of interest is intensively in connection with the filed's temperature, the direction-oriented coupling strengths and the magnetic field. It turns out that the stronger coupling strengths and the smaller magnetic field would induce the smaller measurement's uncertainty of interest within the current spin model. Interestingly, we reveal that the evolution of the uncertainty exhibits quite different dynamical behaviors in antiferromagnetic ($\boldsymbol{J}_i > 0$) and ferromagnetic ($\boldsymbol{J}_i < 0$) frames. Besides, an analytical solution related to the systematic entanglement (i.e., concurrence) is also derived in such a scenario. Furthermore, it is found that the DM-interaction is desirably working to diminish the magnitude of the measurement's uncertainty in the region of high-temperature. Finally, we remarkably offer a resultful strategy to govern the entropy-based uncertainty through utilizing quantum weak measurements, being of fundamentally importance to quantum measurement estimation in the context of solid-state-based quantum information processing and computation.

Keywords measurement uncertainty, concurrence, Heisenberg XYZ chain, weak measurement, lower bound

1 Introduction

Heisenberg uncertainty principle presented in 1927 is generally acknowledged as a distinctive feature of quantum mechanics differing from the classical physics, and it provides an essential restriction to the precision of the measurement's outcomes for a pair of incompatible observables [1]. Canonically, Heisenberg's uncertainty relation can be expressed as: $\Delta x \Delta p_x \geq \hbar/2$ [2] for position x and momentum p_x [3, 4]. Before long, Kennard [4] and Robertson [5] put forward a generalization of the relation into a so-called standard deviation [6, 7]

$$\Delta R \cdot \Delta Q \geq \frac{1}{2} \left| \langle [R, Q] \rangle \right|, \tag{1}$$

for two arbitrary non-commuting observables R and Q. Noting that, $[R, Q] = RQ - QR$ stands for the commutator. Due to that the lower bound in Eq. (1) is dependent of the systematic state, the well-known deviation in essence is not an optimal quantification way regarding the measurement's uncertainty. To improve the methods for surveying the uncertainty of interest, Kraus had suggested a simplified description in terms of the concept of Shannon entropy [8] and later strengthened by Maassen and Uffink [9] into alleged entropic uncertainty relation as

$$H(R) + H(Q) \geq -\log_2 c(R, Q), \tag{2}$$

where the Shannon entropy $H(Y) = -\sum_i p_i \log_2 p_i$ with operator $Y \in \{R, Q\}$ and $p_i = \langle y_i|\rho|y_i\rangle$ in a quantum system ρ, which indicates the Shannon entropy of the probability distribution of the outcomes when Y is measured [10–12]. Note that, $|y_i\rangle$ stand for the eigenvectors of the observables Y. Besides, $-\log_2 c(R, Q)$ shows the complementarity of R and Q, and $c(R, Q) = \max_{i,j} |\langle \varphi_i|\phi_j\rangle|^2$, where $|\varphi_i\rangle$ and $|\phi_j\rangle$ denote the eigenvectors of R and Q, respectively.

In recent years, a new expression of uncertainty relation [13, 14] has been presented when the quantum entanglement appears, and demonstrated by some promising experiments [15–17]. Generally, this relation could be elaborated by a uncertainty game within a couple of legitimate participants (say, Alice and Bob). At start, there have two entangled particles A and B in Bob's site. Bob then delivers A to Alice via quantum channel, and subsequently Alice shall choose one of R and Q to perform a measurement on A. Finally, Alice sends a classical message to Bob to tell which measurement she chooses. As a result, such actions will enable Bob to predict A's measurement outcome under a minimal declination. In this scenario, particle A that is sent to Alice is denominated as the measured system. Contrarily, we denominate particle B as memory system. Quantitatively, the Bob's uncertainty about the result of measurement can be provided by entanglement-assisted entropic uncertainty relation (EUR) [14], viz.,

$$S(R|B) + S(Q|B) \geq S(A|B) - \log_2 c(R, Q), \tag{3}$$

with the above, $S(A|B) = S(\rho_{AB}) - S(\rho_B)$ [2] is a conditional von Neumann entropy. Towards the left item of Eq. (3), $S(Y|B) = S(\rho_{YB}) - S(\rho_B)$ stands for the uncertainty of the measurement outcomes of Y conditioned on the underlying information reserved in particle B. From Eq. (3), one can realize that, Bob can perfectly predict Alice's measuring outcomes if A and B are maximally entangled owing to $S(A|B) = \log_2 c$. In addition, $S(\rho_{YB}) = -\sum_i \lambda_i \log_2 \lambda_i$ [18, 19] with $\rho_{YB} = \sum_j (|y_j\rangle\langle y_j| \otimes I)\rho_{AB}(|y_j\rangle\langle y_j| \otimes I)$, where λ_i are denoted as the eigenvalues of ρ_{YB}, and $|y_j\rangle$ are the eigenstates of Y.

Amazingly, entanglement-assisted EUR has burst out many potential applications in the realm of quantum information science, including quantum metrology [20, 21], entanglement witnessing [14–16, 22–24], quantum transition [25, 26], quantum key distribution [27, 28], cryptography [29, 30] and quantum randomness [31, 32]. Driven by its appealing performance, the entropic uncertainty relation had received much attention by a number of authors. Explicitly, several tighter bounds for EUR had been originally put forward by [33, 34], and the evolution's characteristic of EUR in various realistic environments [35–47] had been reported.

During various solid-state systems, Heisenberg spin chain is usually deemed as one of remarkable and versatile systems due to well-scalable features during quantum information processing [48–50]. With this in mind, clarifying how the measurement uncertainty evolves in Heisenberg spin models is a basic question in the context of quantum measurement estimation. In this article, we will focus on exploring the dynamic of EUR for a two-spin Heisenberg XYZ model under a magnetic field and a canonical Dzyaloshinskii–Moriya (DM) interaction.

The remainder of this article is arranged as follows. In Section 2, we will firstly introduce the theoretical model under a Heisenberg XYZ chain with an external magnetic field and x-direction DM interaction. Specifically, we detailedly discuss the effect of the magnetic field and DM-interaction on entropic uncertainty and derive the dynamic of the systematic entanglement in Section 3. We furthermore put forward a method to steer the measurement's uncertainty by means of quantum weak measurement in Section 4. Finally, a concise conclusion will be given in Section 5.

2 Theoretical model

Let us recall a two-qubit spin-1/2 Heisenberg XYZ mode. Technically, its Hamiltonian in the presence of x-component Dzyaloshinskii–Moriya interaction D_x and inhomogeneous magnetic fields can be written as [51]

$$\begin{aligned}\mathcal{H} &= J_x\sigma_1^x\sigma_2^x + J_y\sigma_1^y\sigma_2^y + J_z\sigma_1^z\sigma_2^z + D_x(\sigma_1^y\sigma_2^z - \sigma_1^z\sigma_2^y) \\ &\quad + (M+m)\sigma_1^x + (M-m)\sigma_2^x,\end{aligned} \tag{4}$$

where J_i are real-valued coupling strengths and σ_k^i ($i = x, y, z$) are denoted as spin-1/2 Pauli operators in the Hilbert space of the k-th qubit, respectively. Canonically, with respect to the cases of $J_x = J_y = J_z$, $J_x = J_y \neq J_z$ and $J_x \neq J_y \neq J_z$, the Heisenberg chains can be defined by XXX, XXZ and XYZ models. Meanwhile, $J_i > 0$ means the antiferromagnetic regime, and $J_i < 0$ means the ferromagnetic one. M stands for the strength of the magnetic field, and m represents the magnitude of the field's inhomogeneity.

At a thermal equilibrium, the density matrix of a two-qubit anisotropic Heisenberg XYZ chain system can be expressed by [51]

$$\rho(T) = \frac{1}{2\mathcal{N}}\begin{pmatrix} L_+ & G_+^* & G_-^* & L_- \\ G_+ & F_+ & F_- & G_- \\ G_- & F_- & F_+ & G_+ \\ L_- & G_-^* & G_+^* & L_+ \end{pmatrix}, \tag{5}$$

where

$$\mathcal{N} = 2\left(\mathrm{e}^{\frac{-J_x}{T}}\cosh\frac{\omega_1}{T} + \mathrm{e}^{\frac{J_x}{T}}\cosh\frac{\omega_2}{T}\right), \tag{6}$$

$$L_\pm = \mathrm{e}^{-\frac{J_x+\omega_1}{T}}\sin^2\phi_1 + \mathrm{e}^{-\frac{J_x-\omega_1}{T}}\sin^2\phi_2 \pm \mathrm{e}^{\frac{J_x-\omega_2}{T}}\sin^2\phi_3 \pm \mathrm{e}^{\frac{J_x+\omega_2}{T}}\sin^2\phi_4, \tag{7}$$

$$F_\pm = \mathrm{e}^{-\frac{J_x+\omega_1}{T}}\cos^2\phi_1 + \mathrm{e}^{-\frac{J_x-\omega_1}{T}}\cos^2\phi_2 \pm \mathrm{e}^{\frac{J_x-\omega_2}{T}}\cos^2\phi_3 \pm \mathrm{e}^{\frac{J_x+\omega_2}{T}}\cos^2\phi_4, \tag{8}$$

$$G_\pm = \mathrm{e}^{-\frac{J_x+\omega_1}{T}}\sin\phi_1\cos\phi_1 + \mathrm{e}^{-\frac{J_x-\omega_1}{T}}\sin\phi_2\cos\phi_2 \pm \mathrm{e}^{\frac{J_x-\omega_2}{T}}\chi\sin\phi_3\cos\phi_3 \pm \mathrm{e}^{\frac{J_x+\omega_2}{T}}\chi\sin\phi_4\cos\phi_4, \tag{9}$$

with $\phi_{1,2} = \arctan\frac{2M}{J_y-J_z\pm\omega_1}$, $\phi_{3,4} = \arctan\frac{2\sqrt{m^2+D_x^2}}{-J_y-J_z\pm\omega_2}$, $\chi = \frac{-\mathrm{i}D_x-m}{\sqrt{m^2+D_x^2}}$, $\omega_1 = \sqrt{(J_y-J_z)^2+4M^2}$, and $\omega_2 = \sqrt{(J_y+J_z)^2+4D_x^2+4m^2}$. Where, T represents the temperature of the field in units of the Boltzmann constant k_B.

To probe the uncertainty in such a model, we take into account a two-spin system with an initial state $|\Psi\rangle_{in} = \cos\frac{\theta}{2}|10\rangle + \mathrm{e}^{\mathrm{i}\phi}\sin\frac{\theta}{2}|01\rangle$ with $\theta \in [0, \pi]$, and $\phi \in [0, 2\pi]$. Based on performing a joint measurement and unitary transformation on the input system we can obtain the final state as the form of [51, 52]

$$\rho_{out} = \sum_{j,k} p_{j,k}(\sigma_j \otimes \sigma_k)\rho_{in}(\sigma_j \otimes \sigma_k), \tag{10}$$

where σ_j $(j = 0, x, y, z)$ represents the identity matrix I and Pauli matrices respectively, and

$$p_{j,k} = \mathrm{Tr}[\mathcal{E}_j\rho(T)]\mathrm{Tr}[\mathcal{E}_k\rho(T)] \tag{11}$$

with $\sum p_{j,k} = 1$ and $\rho_{in} = |\Psi\rangle_{in}\langle\Psi|$. Within the above, $\mathcal{E}_0 = |\varphi^-\rangle\langle\varphi^-|$, $\mathcal{E}_x = |\psi^-\rangle\langle\psi^-|$, $\mathcal{E}_y = |\psi^+\rangle\langle\psi^+|$ and $\mathcal{E}_z = |\varphi^+\rangle\langle\varphi^+|$ with $|\varphi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ and $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$.

Through calculating Eq. (10), we thus obtain the output state as

$$\rho_{AB}=\frac{1}{\mathcal{N}^2}\begin{pmatrix} L_+F_+ & 0 & 0 & \dfrac{\sin\theta L_-F_-}{\cos\phi-\mathrm{i}\cos\theta\sin\phi} \\ 0 & \dfrac{\mathrm{e}^{-\mathrm{i}\phi}[(1+\cos\theta)L_+^2-\mathrm{e}^{2\mathrm{i}\phi}(\cos\theta-1)F_+^2]}{2(\cos\phi-\mathrm{i}\cos\theta\sin\phi)} & \dfrac{\sin\theta(L_-^2+F_-^2)}{2(\cos\phi-\mathrm{i}\cos\theta\sin\phi)} & 0 \\ 0 & \dfrac{\sin\theta(L_-^2+F_-^2)}{2(\cos\phi-\mathrm{i}\cos\theta\sin\phi)} & \dfrac{\mathrm{e}^{-\mathrm{i}\phi}[-\mathrm{e}^{2\mathrm{i}\phi}(\cos\theta-1)L_+^2+(1+\cos\theta)F_+^2]}{2(\cos\phi-\mathrm{i}\cos\theta\sin\phi)} & 0 \\ \dfrac{\sin\theta L_-F_-}{\cos\phi-\mathrm{i}\cos\theta\sin\phi} & 0 & 0 & L_+F_+ \end{pmatrix}, \tag{12}$$

which belongs to an ensemble of states with X-structure density matrices, hereafter ρ_{ij} represent the elements of the density matrix ρ_{AB}.

3 Effect of inhomogeneous magnetic field and DM-interaction on entropic uncertainty relation

In this section, let us focus on investigating how the external magnetic field and the DM-interaction collectively work on the measurement's uncertainty in the current consideration. To do so, we can resort to σ_x and σ_z as the two incompatibility. Canonically, an arbitrary two-qubit state with X-structure density matrix can be described by

$$\rho_{AB}^X = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix}. \tag{13}$$

Thereby, we could derive two post-measurement states $\rho_{\sigma_x}B$ and $\rho_{\sigma_z B}$ for the conditional von Neumann entropies in Eq. (3) as

$$\begin{aligned}\rho_{\sigma_x B} = {}& \frac{\rho_{11}+\rho_{33}}{2}(|00\rangle\langle 00| + |10\rangle\langle 10|) \\ &+ \frac{\rho_{22}+\rho_{44}}{2}(|01\rangle\langle 01| + |11\rangle\langle 11|) \\ &+ \frac{\rho_{14}+\rho_{32}}{2}(|00\rangle\langle 11| + |10\rangle\langle 01|) \\ &+ \frac{\rho_{41}+\rho_{23}}{2}(|01\rangle\langle 10| + |11\rangle\langle 00|),\end{aligned}$$

$$\begin{aligned}\rho_{\sigma_z B} = {}& \rho_{11}|00\rangle\langle 00) + \rho_{22}|01\rangle\langle 01| \\ &+ \rho_{33}|10\rangle\langle 10| + \rho_{44}|11\rangle\langle 11|,\end{aligned} \tag{14}$$

respectively.

Then the von Neumann entropies would be given as

$$S(\rho_{\sigma_x B}) = H_{bin}\left(\frac{1-\sqrt{1-4\eta}}{2}\right) + 1, \tag{15}$$

$$S(\rho_{\sigma_z B}) = -\sum_i \rho_{ii}\log_2\rho_{ii}, \tag{16}$$

where the binary entropy denotes $H_{bin}(\xi) = -\xi\log_2\xi - (1-\xi)\log_2(1-\xi)$ and $\eta = \rho_{11}\rho_{22} + \rho_{22}\rho_{33} + \rho_{33}\rho_{44} + \rho_{44}\rho_{11} - \rho_{14}\rho_{41} - \rho_{32}\rho_{41} - \rho_{23}\rho_{14} - \rho_{23}\rho_{32}$.

Due to $\rho_B = \mathrm{Tr}_A(\rho_{AB})$, one easily attains the eigenvalues of B's reduced density matrix as $\rho_{11}+\rho_{33}$ and $\rho_{22}+\rho_{44}$, the left-hand side (LHS) of Eq. (3) becomes

$$U_L = S(\rho_{\sigma_x B}) + S(\rho_{\sigma_z B}) - 2H_{bin}(\rho_{11}+\rho_{33}). \tag{17}$$

Revisiting the density matrix in Eq. (13), we can derive the eigenvalues of ρ_{AB} as $\lambda_{1,2}^{AB} = \frac{1}{2}(\omega \pm \sqrt{\omega^2 - 4\rho_{11}\rho_{44} + 4\rho_{14}\rho_{41}})$ and $\lambda_{3,4}^{AB} = \frac{1}{2}(\upsilon \pm \sqrt{\upsilon^2 - 4\rho_{22}\rho_{33} + 4\rho_{23}\rho_{32}})$ with $\omega = \rho_{11} + \rho_{44}$ and $\upsilon = \rho_{22} + \rho_{33}$. Meanwhile, the overlap c of two Pauli observables always equals to $\frac{1}{2}$. Therefore, the lower bound in Eq. (3) can be analytically expressed by

$$U_R = 1 - H_{bin}(\rho_{11}+\rho_{33}) - \sum_i \lambda_i^{AB}\log_2\lambda_i^{AB}. \tag{18}$$

Through substituting Eq. (12) into Eqs. (17) and (18), we can derive the exact expressions of U_L and U_R in the current scenario considered here, respectively.

Firstly, we focus on how the field temperature T influences the measurement's uncertainty and its bound. To do so, we depict the uncertainty of interest and the uncertainty's bound in Eq. (3) as functions of T for the magnetic fields with inhomogeneity $m = 0.2$ and $m = 1.0$ respectively, as shown in Fig. 1. Following the graphs, it is clear to show that the entropic uncertainty and lower bound is going to inflate with the growing temperature T and asymptotically tend into a fixed value, showing $U_L \geq U_R$ holds. It displays that the smaller temperature can induce the smaller uncertainty in our considered system. Note that, the uncertainty of interest is closely equivalent to the bound U_R in the regions of relatively high-temperature and low-temperature. In this sense, we can say that the bound can intrinsically and explicitly reflect the measurement uncertainty in those referred regions.

Additionally, we also examine the systematic dynamic of entanglement, which is quantified by concurrence [53]. Typically, the concurrence of an arbitrary two-qubit state ρ_{AB} can be written as

$$C(\rho_{AB}) = \max\{0, \sqrt{\varepsilon_1} - \sqrt{\varepsilon_2} - \sqrt{\varepsilon_3} - \sqrt{\varepsilon_4}\}, \tag{19}$$

where, ε_i $(i = 1, 2, 3, 4)$ represent the decreasing-order eigenvalues of the density matrix $\rho_{AB}(\sigma_A^y \otimes \sigma_B^y)\rho_{AB}^*(\sigma_A^y \otimes \sigma_B^y)$. For simplicity, we offer the form of the concurrence with respect to the class of states in Eq. (13), which can be written as $C(\rho_{AB}) = 2\max\{0, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\}$. As a result, we can exactly derive the concurrence in our considered system as

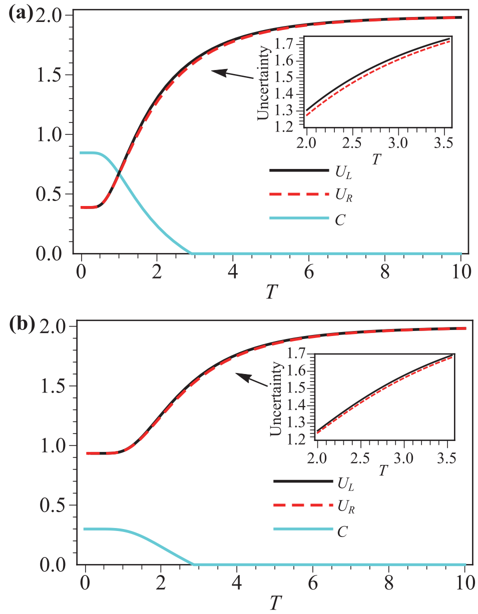

Fig. 1 Entropic uncertainty, concurrence (C) versus the temperature T. Noting that, U_L stands for the left-hand side of (3) and U_R represents the right-hand side of (3). In the Graphs, the black solid lines describe U_L, the red dash-dotted lines describe U_R and the blue solid lines describe the system's concurrence. **(a)** $m = 0.2$ and **(b)** $m = 1.0$. For all plotted with $J_x = 13/3$, $J_y = -7/3$, $J_z = 1.0$, $M = 1.0$, $D_x = 0.2$, $\theta = \pi/2$ and $\phi = 0$.

$$\begin{aligned} C(\rho_{out}) = 2\max\Bigg\{0, &\left|\frac{\sin\theta L_- F_-}{(\cos\phi - \mathrm{i}\cos\theta\sin\phi)\mathcal{N}^2}\right| - \frac{\mathrm{e}^{-\mathrm{i}\phi}\sqrt{((1+\cos\theta)L_+^2 - \mathrm{e}^{2\mathrm{i}\phi}(\cos\theta-1)F_+^2)((1+\cos\theta)F_+^2 - \mathrm{e}^{2\mathrm{i}\phi}(\cos\theta-1)L_+^2)}}{2(\cos\phi - \mathrm{i}\cos\theta\sin\phi)\mathcal{N}^2}, \\ &\left|\frac{\sin\theta(L_-^2 + F_-^2)}{2(\cos\phi - \mathrm{i}\cos\theta\sin\phi)\mathcal{N}^2}\right| - \left|\frac{L_+F_+}{\mathcal{N}^2}\right|\Bigg\}. \end{aligned} \tag{20}$$

Following Fig. 1, the higher temperature will lead to the smaller entanglement (C), and vice versa. With the above analysis in mind, one can realize that the variation trend of the concurrence is considerably opposite to those of both the uncertainty and the bound with growth of T. Based on comparing Figs. 1(a) and (b), we obtain that the stronger inhomogeneity m of the magnetic field will weaken the entanglement between A and B.

We now shift our focus toward probing how the coupling strengths between two-spin affect the measured uncertainty and the lower bound. As shown in Fig. 2, when the system is lying in the case of $J_z > 0$ (antiferromagnetic), we can see that the uncertainty to be probed monotonously decreases with J_z growing, as plotted in Fig. 2(b). In sharp comparison with the antiferromagnetic regime, the evolution of the measurement's uncertainty is not monotonous when $J_z < 0$ in Fig. 2(a). To be explicit, the uncertainty will firstly increase and subsequently decrease with the increasing $|J_z|$. It is worth to noting that, the uncertainty will saturate into a fixed value as the absolute value of J_z becomes large enough. With the different dynamical phenomena of the uncertainty, it can be explained from the viewpoint of the systematic entanglement (concurrence). In the ferromagnetic regime, the entanglement will first reduce and gradually recover up to a maxima with the growing J_z, this naturally leads to that the uncertainty will first inflate and then decrease up to a fixed value. Contrarily, in the antiferromagnetic case, the entanglement will be lift always with the growing J_z, which will definitely result in the monotonic reduction of the uncertainty. Interestingly, U_L is close to U_R all the time, which implies the Berta's bound is a good quantifier to predict the measurement outcome in these cases mentioned above. Furthermore, we can see the concurrence is nearly anti-correlated to the uncertainty relation, which is pretty compatible with the statement we made before.

With regard to external magnetic field, it is required to clarify how its effect on the uncertainty is. To do so, we draw the uncertainty as functions of the M and m as displayed in Fig. 3. Following the figure, it is not difficult

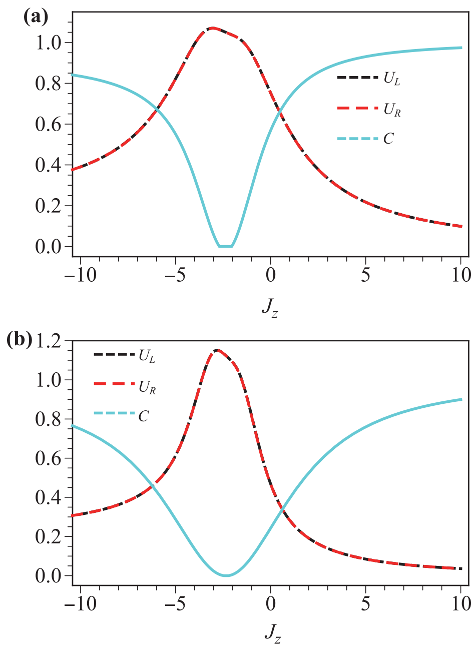

Fig. 2 Entropic uncertainty, concurrence (C) as a function of J_z. The black solid lines plot U_L, red dash-dotted lines plot U_R and blue solid lines plot the concurrence. Noting that, U_L stands for the left item of Eq. (3) and U_R represents the right item of Eq. (3). Here, **(a)** $J_x = -13/3$ and $J_y = -7/3$; **(b)** $J_x = 13/3$ and $J_y = 7/3$. For all plotted with $m = 0.5$, $M = 1.0$, $D_x = 0.2$, $T = 1.0$, $\theta = \pi/2$ and $\phi = 0$.

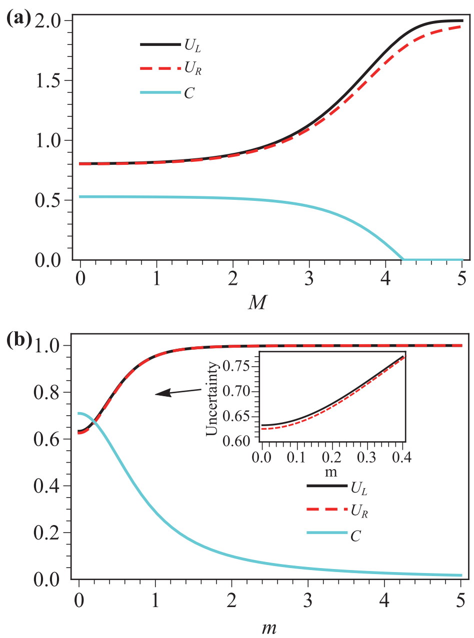

Fig. 3 Entropic uncertainty, concurrence (C) versus the magnetic field M and the inhomogeneity m respectively. In this figure, black solid lines depict U_L, red dash-dotted lines depict U_R and blue solid lines depict the concurrence. Noting that, U_L stands for the left item of Eq. (3) and U_R represents the right item of Eq. (3). For all, $J_x = \frac{13}{3}$, $J_y = -\frac{7}{3}$, $J_z = 1.0$, $D_x = 0.2$, $T = 1.0$, $\theta = \frac{\pi}{2}$ and $\phi = 0$. Additionally, in **(a)** we have $m = 0.5$, and $M = 1.0$ is chosen in **(b)**.

to find that the uncertainty monotonously boosts with the growth of both the strength of external magnetic field M and the degree of inhomogeneous field m. That is to say, the smaller M and m can lead to smaller uncertainty for the measurement, which in essence matches with our conclusions made before according to Fig. 1. On balance, either stronger external magnetic field and larger inhomogeneity will destroy the entanglement, this inevitably results in the increase of the measurement's uncertainty.

Particularly, we further examine the dynamical traits of the measurement's uncertainty in the presence of x-direction DM interaction. For the varying DM interaction strength D_x, we find that the effect of D_x on the uncertainty relation has positive and negative aspects respectively for different temperature regions as shown in Fig. 4(a). It is clear that the stronger DM-interaction is able to distinctly reduce the uncertainty via entropy in the high-temperature regions. Next, let us turn to discuss the case in the region of low-temperature, and it can be seen that the uncertainty increases with the growth of D_x. In principle, a spin system is relatively hard to prepare in an extreme low-temperature in reality. As a result, we can

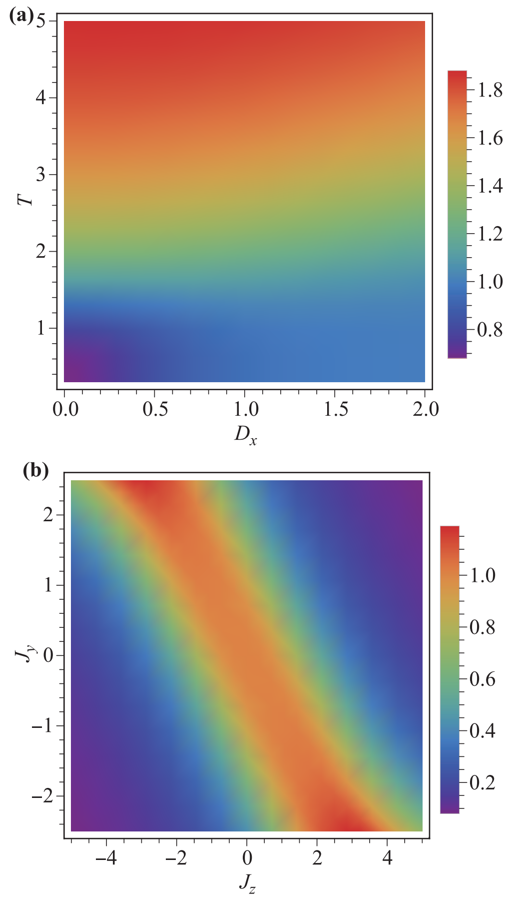

Fig. 4 **(a)** The dynamic of EUR with respect to T and D_x. Here, we take $J_z = 1.0$ and $J_y = -7/3$. **(b)** The dynamic of EUR with respect to J_y and J_z. Here, we take $T = 1.0$ and $D_x = 0.2$. For all plots, we take $J_x = 13/3$, $M = 1.0$, $m = 0.5$, $\theta = \pi/2$ and $\phi = 0$.

say the DM-interaction is an alternative and valid method to reduce the magnitude of the uncertainty in the current scenario. From Fig. 4(b), it also shows that the uncertainty has different variation tendency in antiferromagnetic case ($J_i > 0$) and ferromagnetic case ($J_i < 0$) respectively, as stated in Fig. 2.

4 To reduce the measurement's uncertainty with adopting a local non-unitary operation

Because a quantum system is naturally exposed to its surrounding reservoirs, it unavoidably interacts with the environment noises. Thereby, a quantum system will give rise to quantum decoherence and dissipative effects. To suppress attenuation of an ideal quantum system, which is recognized as an important cornerstone in quantum precision measurement, Aharonov [54] has put forward a novel strategy called quantum weak measurement (WM) [54–60]. Enlighten by this idea, we would like to ask if such an operation is effective to degrade the measurement uncertainty when enhancing the system's robustness. Fortunately, our investigation has proofed that the answer is positive. In the following, we will discuss how this operation influences on our concern in details.

First of all, let us briefly recall quantum weak measurement, which is usually mapped into the following operator

$$\mathcal{K}_\alpha = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-r_\alpha} \end{pmatrix}. \tag{21}$$

This operator is non-unitary, where $\mathcal{K}_\alpha$ represents the operator acting on particle α ($\alpha \in \{A, B\}$) and the measurement strength $r_\alpha \in [0,1]$. Herein, we assume that such operations are performed on both A and B. After that, the system's state can be described by

$$\rho_{AB}^{\mathcal{K}} = \frac{(\mathcal{K}_A \otimes \mathcal{K}_B)\rho_{AB}(\mathcal{K}_A \otimes \mathcal{K}_B)^\dagger}{\mathrm{Tr}\left[(\mathcal{K}_A \otimes \mathcal{K}_B)\rho_{AB}(\mathcal{K}_A \otimes \mathcal{K}_B)^\dagger\right]}, \tag{22}$$

which can be rewritten by a density matrix $\rho_{AB}^{\mathcal{K}}(t)$ of form

$$\begin{aligned}\rho_{AB}^{\mathcal{K}}(t) = {} & \frac{L_+F_+}{\Delta}\left|00\right\rangle\left\langle 00\right| + \frac{\mathrm{e}^{-\mathrm{i}\phi}(1-r_B)\left[L_+^2(\cos\theta+1) - \mathrm{e}^{2\mathrm{i}\phi}F_+^2(\cos\theta-1)\right]}{2\Delta(\cos\phi - \mathrm{i}\cos\theta\sin\phi)}\left|01\right\rangle\left\langle 01\right| \\ & + \frac{\mathrm{e}^{-\mathrm{i}\phi}(1-r_A)\left[F_+^2(\cos\theta+1) - L_+^2\mathrm{e}^{2\mathrm{i}\phi}(\cos\theta-1)\right]}{2\Delta(\cos\phi - \mathrm{i}\cos\theta\sin\phi)}\left|10\right\rangle\left\langle 10\right| + \frac{(1-r_A)(1-r_B)L_+F_+}{\Delta}\left|11\right\rangle\left\langle 11\right| \\ & + \frac{\sin\theta\sqrt{1-r_A}\sqrt{1-r_B}}{2\Delta(\cos\phi - \mathrm{i}\cos\theta\sin\phi)}\left[2L_-F_-(\left|00\right\rangle\left\langle 11\right| + \left|11\right\rangle\left\langle 00\right|) + \left(L_-^2 + F_-^2\right)(\left|01\right\rangle\left\langle 10\right| + \left|10\right\rangle\left\langle 01\right|)\right]\end{aligned} \tag{23}$$

with

$$\begin{aligned}\Delta = {} & L_+F_+ + L_+F_+(1-r_A)(1-r_B) + \frac{\mathrm{e}^{-\mathrm{i}\phi}(1-r_A)\left[F_+^2(\cos\theta+1) - L_+^2\mathrm{e}^{2\mathrm{i}\phi}(\cos\theta-1)\right]}{2(\cos\phi - \mathrm{i}\cos\theta\sin\phi)} \\ & + \frac{\mathrm{e}^{-\mathrm{i}\phi}(1-r_B)\left[L_+^2(\cos\theta+1) - \mathrm{e}^{2\mathrm{i}\phi}F_+^2(\cos\theta-1)\right]}{2(\cos\phi - \mathrm{i}\cos\theta\sin\phi)}.\end{aligned} \tag{24}$$

For convenience, we hereafter take $r_A = r_B = r$ in the current scenario. By linking Eqs. (14)–(18) with Eqs. (23)–(24), the entropic uncertainty relation for the system $\rho_{AB}^{\mathcal{K}}(t)$ to be probed can be obtained analytically.

By means of achieving the local weak measurement on the bipartite AB, one can see the relationship between the measurement's strength r and the uncertainty of interest, as shown in Fig. 5 when M is different with $T = 3.0$ and $T = 4.0$ respectively. In Figs. 5(a) and (b), it is found that the measurement uncertainty validly decreases with the growth of the operational strength r. Meanwhile, we can obtain the temperature also can influence the outcome of the WMR by comparing Figs. 5(a) and (b). In light of the above statements, one can attain that the local weak measurement can effectively degraded the magnitude of the uncertainty. In this sense, we claim the quantum weak measurement is a good candidate for the reduction of the measure uncertainty in our consideration.

It is worth noting that the quantum weak measurements are perfect to achieve our aim in the region of high-temperature, and is effective partly in low-temperature regions, as shown in Fig. 6(a). As to these effective regions, the uncertainty will decrease monotonically when the measurement strength γ increases, which reflects that the WM can perfectly reduce the uncertainty of interest. In general, the state will turn to a mixed state from purity state in the process of a qubit evolving from ground state to excited state, and thus the field will weaken the systematic entanglement between particle A and B. For high-temperature regions, the weak measurement on both A and B will prompt the conversion of from the excited state to the ground state, which will lead to the increase of systematic entanglement (concurrence). By comparison, in the region of low-temperature, it will restrain the probability of the above-mentioned conversion. As a result, the effect of weak measurement is not obvious. From

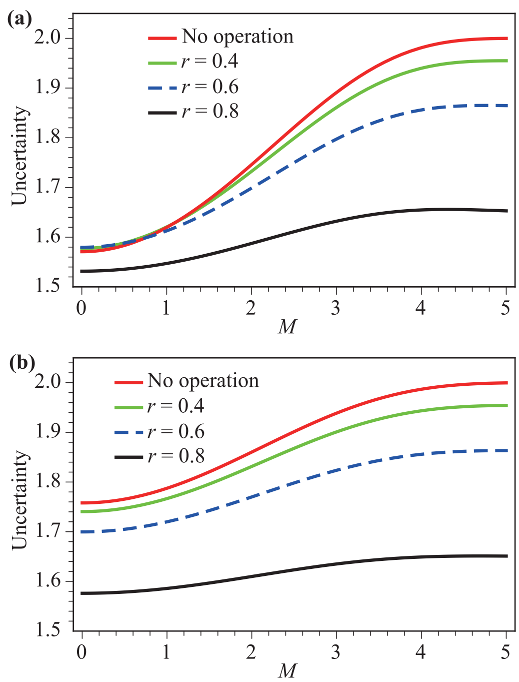

Fig. 5 Measurement uncertainty versus the magnetic field M with regard to different operational strengths r. r ranges from 0 to 0.8 from top to bottom. Here, **(a)** $T = 3.0$ and **(b)** $T = 4.0$. For all plotted with $J_x = 13/3$, $J_y = -7/3$, $D_x = 0.2$, $J_z = 1.0$, $m = 0.5$, $\theta = \pi/2$ and $\phi = 0$.

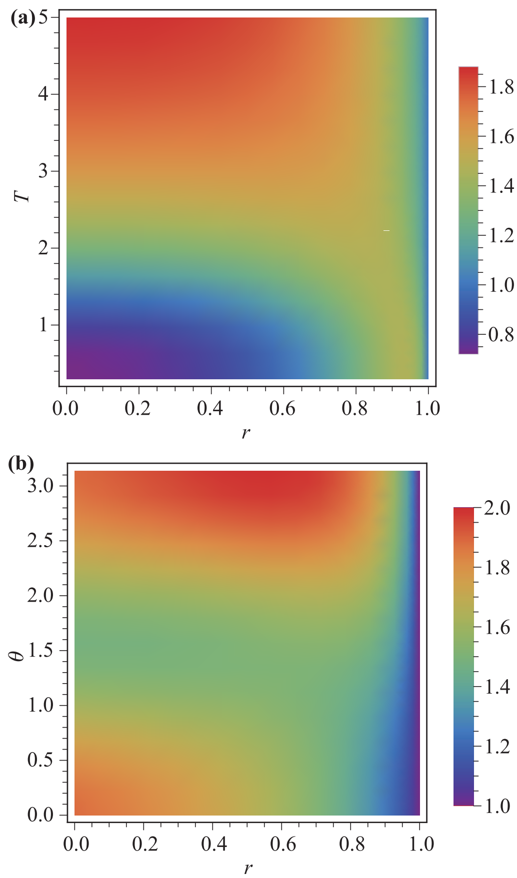

Fig. 6 **(a)** The dynamic of EUR with respect to T and r. The amplitude angle of the initial state $\theta = \pi/2$ is chosen. **(b)** The dynamic of EUR with respect to θ and r. The field temperature is taken as $T = 5/2$. For all plots, $J_x = 13/3$, $J_y = -7/3$, $M = 0.5$, $J_z = 1.0$, $m = 0.5$ and the phase $\phi = 0$.

Fig. 6(b) with $T = 2.5$, we can see that the quantum weak measurement is not effective if $\theta \to \pi$ and the measurement strength is not strong enough. However, we can realize that WM also can validly reduce the uncertainty in the relatively large measurement strength regions, which meets our expectation. Thus, it can be seen that the quantum weak measurements are not effective for the all situations, but the quantum weak measurements are working to reduce the uncertainty of interest in the most cases considered here.

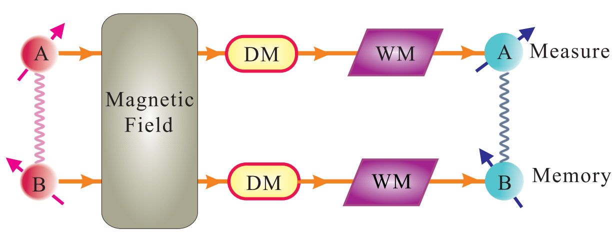

Fig. 7 The schematic diagram for the entire evolution with the time from the left to right in the current scenario. Note that, DM means Dzyaloshinskii–Moriya interaction and WM stands for quantum weak measurement.

5 Conclusion

In summary, we have studied the dynamical characteristic of the measurement's uncertainty with respect to a two-qubit Heisenberg XYZ spin model with the DM interaction in the background of an inhomogeneous magnetic field, as illustrated in Fig. 7. Specially, we take into account the effect of the field temperature T on the entropic uncertainty with different m. We find that the smaller degree of the inhomogeneous field would result in a smaller uncertainty of the measurement in the low-temperature region. And it reveals that the evolution of the uncertainty exhibits quite different dynamical behaviors in antiferromagnetic and ferromagnetic cases. In addition, we analyze the DM-interaction will demonstrate positive and negative influence for different temperature regions. It shows that the DM-interaction is desirably working to effectively reduce the magnitude of the measurement uncertainty in the region of high-temperature. Furthermore, we propose a strategy—quantum weak measurement—to steer the uncertainty, which can validly reduce the uncertainty of interest in the current scenario mostly. Thereby, we argue that our results might benefit to in-depth understanding for the entanglement dynamic of the spin-based solid-state systems, and also impose the illumination of quantum measurement precision in practical quantum information processing.

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