

An improved noise reduction algorithm based on wavelet transformation for MEMS gyroscope

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Abstract To solve the large noise problem for the low-precision gyroscopes in micro-electro mechanical systems (MEMS) of inertial navigation system, an improved noise reduction method, based on the analyses of the fast Fourier transformation (FFT) noise reduction principle and the simple wavelet noise reduction principle, was proposed. Furthermore, the FFT noise reduction method, the simple wavelet noise reduction method and the improved noise reduction method were comparatively analyzed and experimentally verified in the case of the constant rate and dynamic rate. The experimental analysis results showed that the improved noise reduction method had a very good result in the noise reduction of the gyroscope data at different frequencies, and its performance was superior to those of the FFT noise reduction method and the simple wavelet noise reduction method.

Keywords micro-electro mechanical systems (MEMS), gyroscopes, fast Fourier transformation (FFT) noise reduction, wavelet noise reduction

1 Introduction

Micro-electro mechanical systems (MEMS) gyroscope has been widely used in weapon's guidance, inertial navigation system, car industry, biomedical engineering, precision instruments, aerospace, defense technology, mobile communications and other fields, due to the advantages of the low cost, small size, light weight and reliability [1,2]. For MEMS gyro signal processing, fast Fourier transform (FFT) cannot reflect noise signal changes within a certain period of time or a particular point of time. In addition, it cannot distinguish the difference between the high frequency portion of useful

signal and the high frequency portion of noise. If the low pass filter bandwidth is too wide, the noise will be introduced into the useful signal, whereas the signal will be severely distorted [3,4]. The wavelet transform with multi-resolution analysis features can analyze the signal in the time domain and frequency domain simultaneously, which has the ability to characterize local features of the signal. For the wavelet transform, the window size is fixed, but the shape can be changed. When using the wavelet noise reduction, a large amount of the calculation will be increased and that may be less useful to extract the high-frequency signals if the decomposition level is too high. Meanwhile, a lot of the noise will be kept but if the decomposition level is too low. In view of the shortage of the FFT noise reduction method and the simple wavelet transformation noise reduction method [5–7], an improved noise reduction processing methods based on wavelet transform was proposed in this paper, the comparison and analysis with the traditional FFT noise reduction and the simple wavelet transform noise reduction were performed. A large number of the constant and dynamic rates experiments showed that the improved algorithm had more significantly improved the noise reduction than traditional FFT noise reduction and the simple wavelet transform noise reduction.

2 FFT noise reduction principle

Fourier transform is a kind of very important algorithm in the field of digital signal processing. FFT is the fast algorithm of discrete Fourier transform (DFT), which can use the digital computer to quickly work out [8]. The following formula is N points sequence DFT of $x(n)$:

$$x[k] = \sum_{n=0}^{N-1} x_n e^{-i2\pi kn/N}. \quad (1)$$

The DFT calculation is decomposed into the shorter sequence DFT by means of the FFT algorithm using the rotation factor $W_N = e^{-i2\pi/N}$ with the periodic and symmetry features. The algorithm can be divided into two categories [9]: time extraction method and frequency extraction method. At present, the sequence of length $N = 2L$ (L is an integer) is more commonly used.

The essence of FFT noise reduction processing is that the signal waveform is decomposed into the sum of the different frequency sine/cosine waves. The figure after Fourier transform is the signal spectrum, which reflects the relationship of sine wave amplitude and frequency. According to the signal spectrum, the useful signal spectrum is reserved and the noise spectrum is suppressed, then the inverse Fourier transform to the processed spectrum is carried out, and then the noise reduction signal can be obtained.

3 Principle of wavelet transformation noise reduction

The idea of the wavelet transformation noise reduction is the result of the development of Fourier transform noise reduction. Wavelet transformation can be adopted to analysis signal in time domain and frequency domain at the same time, it has the characteristic of multi-resolution analysis which can denote local signal characteristic in time domain and frequency domain simultaneously. Wavelet transformation not only has the advantages of Fourier transform, but also has the transient signal processing ability [10].

Assuming $x(t)$ is a square-integrable function, namely, $x(t) = L \in L^2(R)$. $\psi(t)$ is a function that called basic wavelet or mother wavelet.

$$WT_x(a, \tau) = \frac{1}{\sqrt{a}} \int x(t) \psi^* \left(\frac{t-\tau}{a} \right) dt = \langle x(t), \psi_{a\tau}(t) \rangle. \quad (2)$$

$WT_x(a, \tau)$ is called wavelet transform of $x(t)$, where $a > 0$ is the scale factor corresponding to the frequency information and τ represents shift corresponding to the spatial and temporal information $\psi^*(t)$ ($\psi^*(t)$ is the complex conjugate of $\psi(t)$). The Fourier transform of the mother wavelet satisfies condition [11]:

$$F_\psi(\omega) = \int_{-\infty}^{+\infty} \frac{|\psi(\omega)|^2}{\omega} d\omega < +\infty. \quad (3)$$

Making $\psi(t)$ extension and translation and we can get function families:

$$\psi_{a\tau}(t) = \frac{1}{\sqrt{a}} \psi \left(\frac{t-\tau}{a} \right), \quad \tau \in R, a > 0. \quad (4)$$

$\psi_{a\tau}(t)$ is called wavelet basis function, $WT_x(a, \tau)$ is called wavelet coefficients. Through the inverse Fourier trans-

form, the reconstruct function $x(t)$ can be obtained:

$$x(t) = \frac{1}{F_\psi(\omega)} \iint_R \frac{WT_x(a, \tau) \psi_{a\tau}(t)}{a^2} da d\tau. \quad (5)$$

The useful signals of MEMS gyroscope are usually low frequency signals or some smooth change signals, while the noise signals are usually high frequency signals. Traditional noise reduction process using wavelet transformation can be handled like that the wavelet transform decomposing the signal firstly as described in Refs. [12–14]. According to the principle of wavelet decomposition, the signal can be divided into two segments: both low and high frequencies, which the noise is usually contained in high frequency; Then, threshold is set in the high frequency for processing wavelet coefficients so that the noise in the high frequency portion can be suppressed. Finally, the signal is reconstructed and noise cancellation purpose is to be achieved.

4 Improved noise reduction algorithm based on wavelet transformation

4.1 An improved novel noise reduction algorithm

In this paper, the MEMS gyroscope was used in inertial navigation system, in which useful signal is concentrated in the low frequency phase and randomly changes within the dynamic range of 0–80 Hz. The noise is mainly the random Gaussian white noise. In the practical application of engineering, usually there is not too much available data at a certain moment for noise reduction that will affect the noise features extraction, so that noise reduction is not satisfactory. What's worse, the noise will also change randomly in the practical application. To solve these problems, the traditional FFT noise reduction and simple wavelet transformation noise reduction are not applicable, and this paper presents a new method based on wavelet transform noise reduction to improve the noise reduction effect as the following steps:

1) According to the following formula, the variance of sampling data at a certain moment is calculated, and the time interval is set for 1 s, the MEMS gyroscope data sampling frequency is 1000 Hz.

$$s^2 = \frac{1}{n} [(x_1 - M)^2 + (x_2 - M)^2 + \dots + (x_n - M)^2]. \quad (6)$$

2) Using the variance to real-time forecast data noise, and calculating the biggest amplitude value A_{\max} in frequency domain.

3) Considering wavelet transform's noise reduction performance and computational problems in practical engineering application that db3 wavelet is used and 3 layers are broken down. For simple wavelet transform noise reduction when choosing soft threshold processing,

the small wavelet coefficient is set to zero and the large wavelet coefficients is shrunk, thus maybe the useful signals is missed and dynamic performance changes is greater. Aiming at the problems, considering MEMS inertial navigation system's dynamic range and sampling frequency, the thresholds of layers 1 and 2 are set to zero, then, the estimated amplitude value A_{\max} is used to eliminate the noise which is introduced due to lower the threshold.

4) Reconstructing the signals of wavelet decomposition and predicting value A_{\max} de-noising again according to step (2). That is, the frequency values, which are less than the values of A_{\max} , are automatically eliminated; and the frequency values, which are greater than the values of A_{\max} , are reserved.

4.2 Performance analysis of improved algorithm

MEMS gyroscope as the main sensor of inertial navigation system, the noise reduction performance and real-time of the noise reduction of gyroscope's original data must be taken into account. Wavelet transformation noise reduction has great effect on the system's dynamic. The improved algorithm in this study using the number of lower decomposition can reduce not only the complexity of the algorithm, but also the operation time of the algorithm to improve the real time filtering. Then, the correlation of real-time data in a short time was used to estimate the noise for filtering again so that the improved algorithm has better filtering effect compared with the wavelet transform algorithm.

In this paper, the improved algorithm is aiming at improving every other seconds' data of the MEMS gyroscope, which sampling frequency is 1000 Hz, so in practical engineering applications the improved algorithm can approximate real-time noise reduction processing.

4.3 Test of noise performance

To verify the performance of the improved algorithm, MEMS-based inertial navigation system's experiments were conducted. The MEMS gyroscope data sampling frequency is 1000 Hz and the dynamic range of the inertial navigation system is in 0–80 Hz.

4.3.1 Test of constant rate

MEMS inertial navigation system based on digital signal processing (DSP) was put on the uniaxial rate turntable and the turntable's flat was kept parallel to the earth surface and the measuring axis of the gyroscope was also parallel to the axis of the turntable. Then, the turntable rotation was under a constant rate. The MEMS gyroscope raw data are shown in Fig. 1.

Figure 1 shows that the original signal of the MEMS gyroscope has many burrs, that is to say, the original signal has a lot of interferences.

Figure 2 shows that the effect of noise reduction based on improved wavelet transform is superior to FFT noise reduction and simple wavelet transform noise reduction. Then, the zero bias stability of MEMS gyroscope was studied to further analyze the effect of noise reduction. The stability of the zero bias is that the gyro output around its mean value, also known as the stability of the zero drift, the calculation formula is described as follows [15]:

$$B_s = \frac{1}{K} \left[\frac{1}{(n-1)} \sum_{j=1}^n (F_j - \bar{F})^2 \right]^{1/2}. \quad (7)$$

Through the experiment, the original signal stability of zero bias is 0.0225 deg/s. After wavelet transformation noise reduction and FFT noise reduction, the zero bias stability is 0.0036 and 0.0104 deg/s respectively, and the

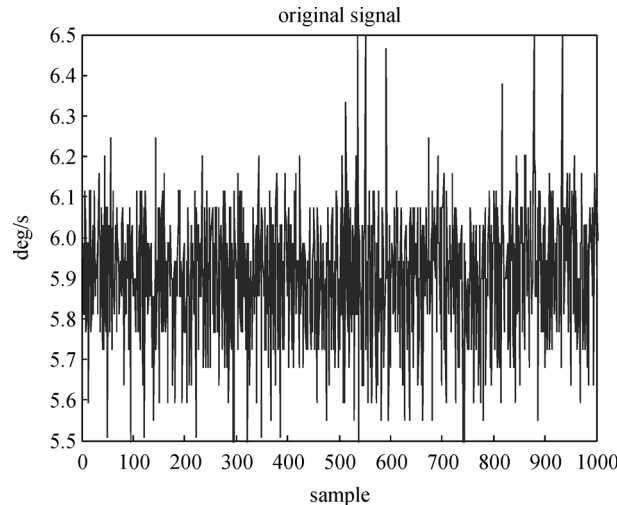


Fig. 1 Original signal of the MEMS gyroscope

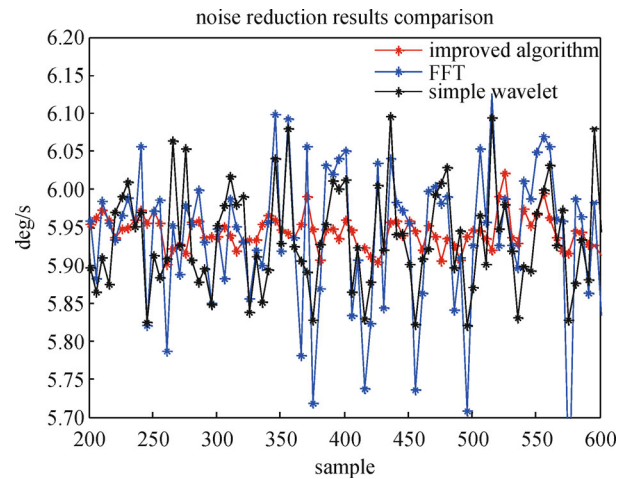


Fig. 2 Comparison results of the constant rate noise reduction

zero bias stability of improved algorithm is 0.0006 deg/s.

4.3.2 Experiment of dynamic rate

Due to lack of equipments, the dynamic rate experiment is based on constant rate experiment, different frequency sine-waves were added to the constant rate data to verify the noise reduction effect of the improved algorithm under

the dynamic condition. With sine wave frequencies of 10, 40 and 75 Hz for example, the sampling frequency is 1000 Hz.

As can be seen from the above Figs. 3, 4 and 5, there are a certain degree of the distortion under different frequencies for both the simple wavelet transformation and the FFT noise reduction. The improved algorithm can better extract the useful signal.

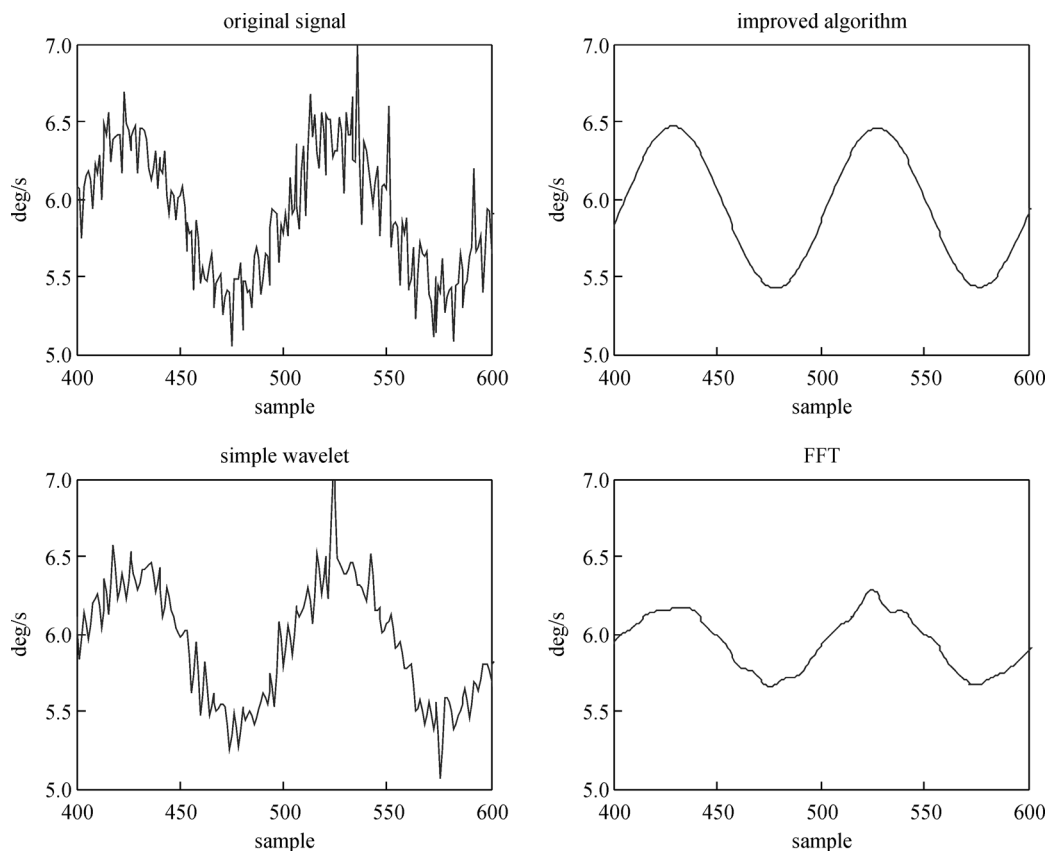


Fig. 3 Comparison of the 10 Hz noise reduction effect

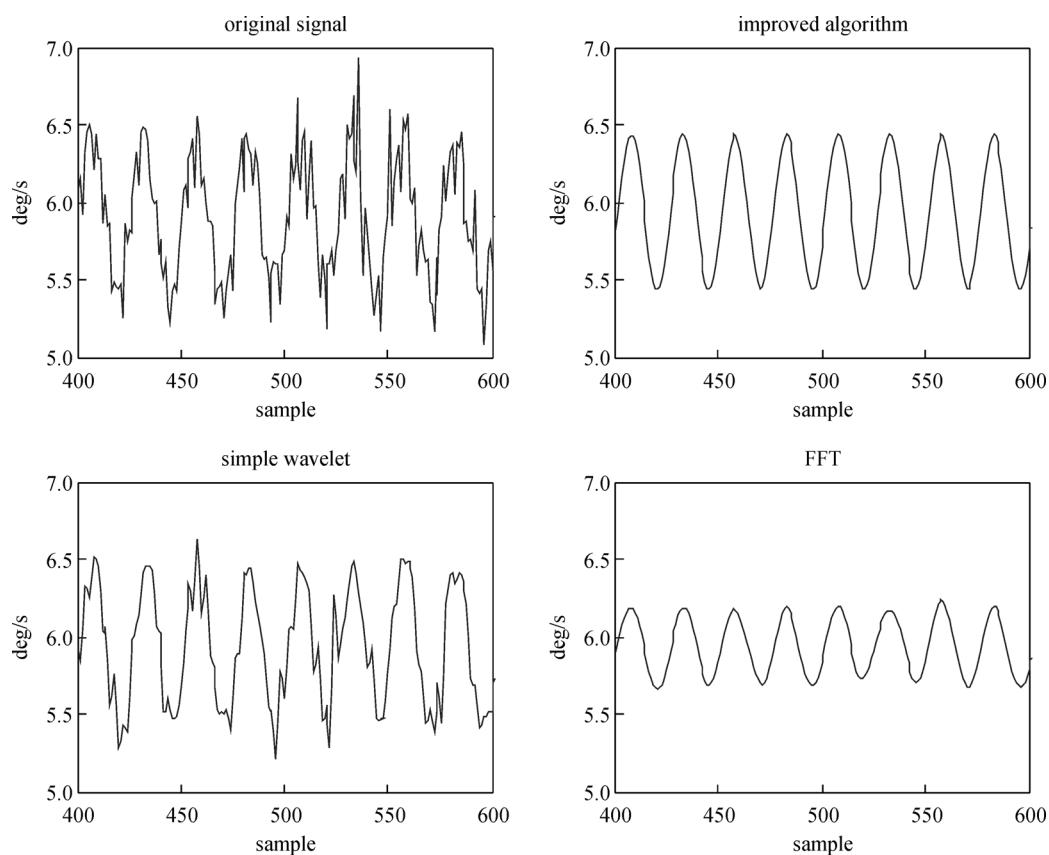


Fig. 4 Comparison of the 40 Hz noise reduction effect

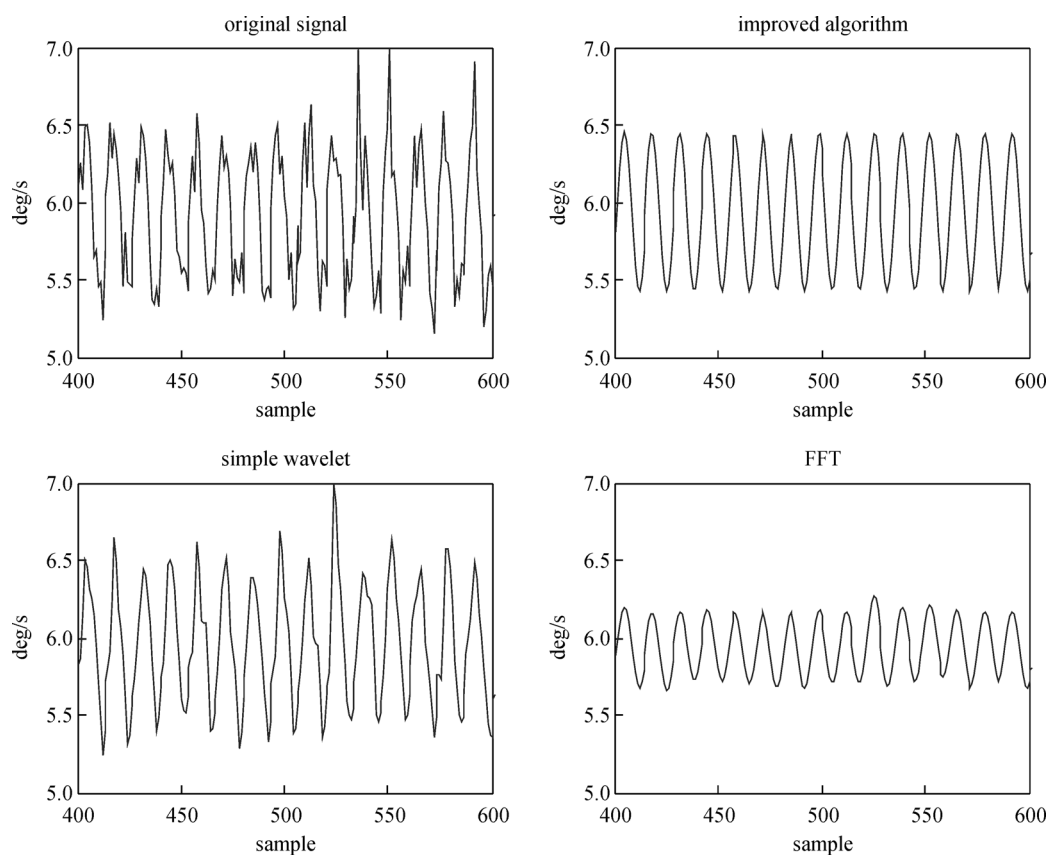


Fig. 5 Comparison of the 75 Hz noise reduction effect

5 Conclusions

Based on the analysis of the simple wavelet transformation noise reduction and the FFT noise reduction principle, this paper proposes an improved noise reduction algorithm based on wavelet transform in view of the low precision and wide range of MEMS gyroscope. The improved algorithm can filter out the noise very well under the condition of without increasing computation. By testing and analysis, it was shown that improved algorithm has a better noise reduction effect in the low dynamic situation or the high dynamic circumstances, and is better than the other two noise reduction methods.

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