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RESEARCH ARTICLE

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Influence factors on natural frequencies of composite materials

Abstract Compared with traditional materials, composite materials have lower specific gravity, larger specific strength, larger specific modulus, and better designability structure and structural performance. However, the variability of structural properties hinders the control and prediction of the performance of composite materials. In this work, the Rayleigh–Ritz and orthogonal polynomial methods were used to derive the dynamic equations of composite materials and obtain the natural frequency expressions on the basis of the constitutive model of laminated composite materials. The correctness of the analytical model was verified by modal hammering and frequency sweep tests. On the basis of the established theoretical model, the influencing factors, including layers, thickness, and fiber angles, on the natural frequencies of laminated composites were analyzed. Furthermore, the coupling effects of layers, fiber angle, and lay-up sequence on the natural frequencies of composites were studied. Research results indicated that the proposed method could accurately and effectively analyze the influence of single and multiple factors on the natural frequencies of composite materials. Hence, this work provides a theoretical basis for preparing composite materials with different natural frequencies and meeting the requirements of different working conditions.

Keywords composite material, hammering and frequency sweep test, structural parameter, natural frequency

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1 Introduction

Fiber-reinforced composite materials are structural composite materials that are widely used in engineering. They are composed of fibers and matrix materials. Fibers perform the main function of materials, and the matrix bonds with fibers to form a whole composite material [1]. In actual engineering, fiber-reinforced composite materials are generally designed as multilayer materials with multiple angles and multiple layers so that the materials can fully exert the bearing effect of fibers in all directions. Differences in angle and layer structures directly affect the natural frequencies of composite materials [2,3]. Therefore, the effects of different structural factors on composite materials need to be studied and analyzed.

In recent years, domestic and foreign scholars have studied the factors affecting the natural frequencies of composite materials by mainly using experimental methods and numerical simulation methods. Norman et al. [4] used the finite element software ANSYS to study the free vibration characteristics of laminated beams under different stacking schemes and different boundary conditions. Imran et al. [5] used the commercial finite element software ABAQUS to study the effect of delamination on the natural frequency of a composite board. Taheri-Behrooz and Pourahmadi [6] used the user-defined Fortran subroutine (USDFLD) to study the nonlinear effect of resin on the mechanical properties of composite materials. Ghasemi et al. [7] established the finite strain equation of a beam to study the influence of beam thickness and different boundary conditions, and the natural frequencies of different composite materials were obtained and compared. Afsharmanesh et al. [8] used the numerical calculation method to study the effects of different boundary conditions on the inherent properties of composite circular plates, as well as the effects of fiber orientation on the natural frequency and critical buckling load of corneal laminates. Ghaheri et al. [9] studied the influence of different classical boundary conditions (free, clamping, and simple support) on the natural frequencies of composite circular plates. In addition, the effect of fiber orientation on the natural frequency and buckling load of laminates was extensively investigated. Ananda Babu and Vasudevan [10] established the differential equation of motion of composite laminates to study the effect of layer thickness on the free and forced vibration response of composite cone plates. Khalid et al. [11] proposed a layer-by-layer equation for the dynamic analysis of multilayer arch structures to study the effect of the number of layers on the natural frequency of a multilayer arch structure. Roque and Martins [12] used a meshless numerical method to study the effect of different stacking orders on the natural frequencies of composite sheets, and the stacking order was optimized. Mukhopadhyay et al. [13] proposed a hybrid high-dimensional model based on uncertain propagation to study the effect of material delamination on the natural frequencies of composite materials. Zhao et al. [14] proposed a unified analytical model to study the free vibration of composite laminated elliptic cylinders with general boundary conditions (including classical boundary, elastic boundary, and their combination). Xue et al. [15] proposed a Fourier series method for the vibration modeling and analysis of composite laminates based on Mindlin theory and Hamilton variational principle, and studied the vibration model of composite laminates under three constrained spring conditions. Singh et al. [16] studied various random characteristics generated during the preparation of composite materials to affect the natural frequencies of materials, and several scholars have conducted targeted research. Leissa and Martin [17] analyzed the effect of variable fiber spacing on the free vibration and buckling of composite sheets. Vigneshwaran and Rajeshkumar [18] used artificial layering methods and compression molding techniques to prepare the composite materials of different matrix materials and studied the effects of different matrix materials on the free vibration characteristics of fiber-reinforced polymer matrix composites. Cevik [19] used the finite element method to study the effects of different layer angles, different boundary conditions, and different aspect ratios on the natural frequencies of composite materials. Zhong et al. [20] used a continuous condition to model to analyze the vibration characteristics of laminated cylindrical thin shells with arbitrary boundary conditions and study the effects of the length ratio of a nonlaminated cylindrical shell and the thickness ratio of the inner and outer layers on the natural frequency and mode. Donadon et al. [21] studied the effect of reinforcing fibers on the mechanical properties of laminated beams through bending tests. Honda and Narita [22] used spline functions represent fibers of arbitrary shapes and serial-type shape functions to derive frequency equations and studied the natural frequencies and mode shapes of composite materials with arbitrary lamination angles. Narita [23] proposed a new layered optimization method, and optimized the vibration characteristics of composite laminates by optimizing the fiber orientation of each layer. Dey et al. [24] used a vector regression (support vector machine regression) model to study the effect of openings on the random natural frequencies of composite laminated curved panels. Djordjević et al. [25] used the finite element modeling to study the influence of fiber orientation angle on the basic dynamic and static characteristics (torsion angle, natural frequency) of composite shaft. Li et al. [26] established a three-layer composite thin cylindrical shell model and studied the effects of boundary conditions, geometric parameters, symmetrical lamination scheme, and damping coefficient on the nonlinear amplitude–frequency characteristics of a symmetric three-layer composite thin cylindrical shell. Fallahi et al. [27] proposed a new modeling theory by reviewing the traditional modeling methods of composite constitutive models and discussed the effects of different factors on the mechanical properties of composites. Wang et al. [28] established a five-degree-of-freedom angular model of contact ball bearing and a complete high-speed dynamic model to analyze and study the influence of nonlinear characteristics on an entire spindle system.

At present, the influence factors on the natural frequencies of laminated composite materials are mainly studied by experimental and simulation methods. The experimental method is generally suitable for single-factor impact analysis and requires a large number of material samples for multiple tests. The simulation method often requires a complex modeling process, and the coupling effect analysis of multiple factors is relatively rare. In the current work, the influence of various factors on the natural frequencies of composite materials was analyzed, and coupling factor influence analysis was conducted. The contribution of this work is not the modeling method; this work particularly provides a detailed discussion about the influences of structural and coupling parameters on the natural frequencies of composite materials on the basis of a theoretical analysis.

2 Dynamic solution of laminated composites

The kinetic energy T of a laminated composite sheet is expressed as

$$T=\frac{\rho h}{2}\int_0^b\int_0^a\left(\frac{\partial w}{\partial t}\right)^2\mathrm{d}x\mathrm{d}y, \tag{1}$$

where a is the width of the composite sheet, b is the length of the composite sheet, h is the thickness of the composite sheet, ρ is the average density of the composite sheet, and w is the disturbance of the composite sheet and is a function of time t, $w = w(x, y, t)$.

As only the plane stress condition of the composite sheet is considered, the strain energy V can be expressed as

$$V=\frac{1}{2}\int_0^b\int_0^a\int_{-\frac{h}{2}}^{\frac{h}{2}}(\sigma_x\varepsilon_x+\sigma_y\varepsilon_y+\tau_{xy}\gamma_{xy})\mathrm{d}z\mathrm{d}x\mathrm{d}y, \quad (2)$$

where σ_x and σ_y are the normal stresses in x and y directions, respectively, τ_{xy} is the corresponding shear stress, ε_x and ε_y are the normal strains in x and y directions, respectively in the middle surface, and γ_{xy} is the shear strain in the middle surface.

Only the deformation caused by bending strain energy is considered here. The strain energy V_2 of the laminated composite sheet can be expressed as

$$\begin{aligned}V_2&=\frac{1}{2}\int_0^b\int_0^a(M_xk_x+M_yk_y+M_{xy}k_{xy})\mathrm{d}x\mathrm{d}y\\&=\frac{1}{2}\int_0^b\int_0^a\left[D_{11}\left(\frac{\partial^2w}{\partial x^2}\right)^2+2D_{12}\frac{\partial^2w}{\partial x^2}\frac{\partial^2w}{\partial y^2}+D_{22}\left(\frac{\partial^2w}{\partial y^2}\right)^2\right.\\&\left.+4D_{16}\frac{\partial^2w}{\partial x^2}\frac{\partial^2w}{\partial y\partial x}+4D_{26}\frac{\partial^2w}{\partial y^2}\frac{\partial^2w}{\partial y\partial x}+4D_{66}\left(\frac{\partial^2w}{\partial y\partial x}\right)^2\right]\mathrm{d}x\mathrm{d}y,\end{aligned} \quad (3)$$

where D_{ij} is the element in the bending (torsion) stiffness matrix of the composite sheet, and the calculation equation is available in Refs. [28–30]. Under the cantilever condition, the specific expression deflection w of the composite sheet can be expressed as

$$w(x,\ y,\ t)=W(x,\ y)\mathrm{e}^{\mathrm{i}\omega t}, \quad (4)$$

where ω is the natural frequency of the laminated composite sheet, and $W(x, y)$ is the modal function of the composite sheet. According to the Rayleigh–Ritz method, the linear combination of shape functions can be used. The expression is shown as

$$W=\sum_{m=1}^{M}\sum_{n=1}^{N}\boldsymbol{q}_{mn}X_m(x)Y_n(y), \quad (5)$$

where $X_m(x)$ and $Y_n(y)$ are orthogonal polynomials under boundary conditions, $m = 1, 2, \ldots, M$, $n = 1, 2, \ldots, N$, M and N are the numbers of selected material nodes, and $\boldsymbol{q}$ is the eigenvector, $\boldsymbol{q} = (q_{11}, q_{12}, q_{13}, \ldots, q_{MN})^{\mathrm{T}}$. The specific representations of each parameter in Eq. (5) can be expressed as

$$\begin{cases}X_1(\xi)=\chi(\xi),\\X_2(\xi)=(\xi-H_2)X_1(\xi),\\X_i(\xi)=(\xi-H_i)X_{i-1}(\xi)-V_iX_{i-2}(\xi),\quad i>2,\end{cases} \quad (6)$$

$$\begin{cases}Y_1(\eta)=\kappa(\eta),\\Y_2(\eta)=(\eta-G_2)Y_1(\eta),\\Y_i(\eta)=(\eta-H_i)Y_{i-1}(\eta)-R_iY_{i-2}(\eta),\ i>2,\end{cases} \quad (7)$$

where H_i, G_i, V_i, and R_i are the coefficient functions, and the specific expressions are shown as follows:

$$\begin{cases}H_i=\dfrac{\int_0^1\phi X_{i-1}^2(\xi)\xi\mathrm{d}\xi}{\int_0^1\phi X_{i-1}^2(\xi)\mathrm{d}\xi},\\V_i=\dfrac{\int_0^1\phi X_{i-1}(\xi)X_{i-2}(\xi)\xi\mathrm{d}\xi}{\int_0^1\phi X_{i-2}^2(\xi)\mathrm{d}\xi},\\\chi(\xi)=\xi^{\alpha}(1-\xi)^{\beta},\\\xi=\dfrac{x}{b},\end{cases} \quad (8)$$

$$\begin{cases}G_i=\dfrac{\int_0^1\phi Y_{i-1}^2(\eta)\eta\mathrm{d}\eta}{\int_0^1\phi Y_{i-1}^2(\eta)\mathrm{d}\eta},\\R_i=\dfrac{\int_0^1\phi Y_{i-1}(\eta)Y_{i-2}(\eta)\eta\mathrm{d}\eta}{\int_0^1\phi Y_{i-2}^2(\eta)\mathrm{d}\eta},\\\kappa(\eta)=\eta^{\gamma}(1-\eta)^{\tau},\\\eta=\dfrac{y}{a},\end{cases} \quad (9)$$

where ϕ is the weight function under the cantilever boundary condition, $\phi = 1$, $\alpha = 2$, $\beta = 0$, $\gamma = 0$, and $\tau = 0$.

For a composite sheet in a cantilever condition, applying the Hamilton principle derives the following expression:

$$\delta\int_{t_1}^{t_2}(V-T)\mathrm{d}t=0, \quad (10)$$

where t_1 and t_2 denote the start and end times of composite sheet vibration, respectively.

By substituting the kinetic energy and strain energy of the composite sheet and the solution form into Eq. (10), in which the kinetic energy and strain energy are maximized, we can obtain the differentiated derivation of eigenvector $\boldsymbol{q}$ as

$$\frac{\partial(V_{\max}-T_{\max})}{\partial\boldsymbol{q}}=0, \quad (11)$$

where $V_{\max}$ is the maximum bending strain energy of the composite sheet and $T_{\max}$ is the maximum kinetic energy of the composite sheet. We substitute the deflection expression Eq. (4) into the maximum kinetic energy Eq. (1) and maximum strain energy Eq. (3). The specific expressions are as follows:

$$T_{\max}=\frac{\rho h}{2}\int_0^b\int_0^aW^2w^2\mathrm{d}x\mathrm{d}y, \quad (12)$$

$$V_{\max}=\frac{1}{2}\int_0^b\int_0^a\left[D_{11}\left(\frac{\partial^2 W}{\partial x^2}\right)^2+2D_{12}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2}+D_{22}\left(\frac{\partial^2 W}{\partial y^2}\right)^2+4D_{16}\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y\partial x}+4D_{26}\frac{\partial^2 W}{\partial y^2}\frac{\partial^2 W}{\partial y\partial x}+4D_{66}\left(\frac{\partial^2 W}{\partial y\partial x}\right)^2\right]\mathrm{d}x\mathrm{d}y. \quad (13)$$

According to Eqs. (12) and (13), the maximum kinetic energy and maximum strain energy of the composite sheet are functions of eigenvector $\boldsymbol{q}$. By substituting Eqs. (12) and (13) into Eq. (10) and then deriving eigenvector $\boldsymbol{q}$, we can obtain Eq. (14), through which the natural frequency of the composite sheet can be calculated.

$$\frac{1}{2}\int_0^a\int_0^b\boldsymbol{D}\begin{Bmatrix} X_1^2 & X_1X_2 & \cdots & X_1X_M \\ X_2X_1 & X_2^2 & \cdots & X_2X_M \\ \cdots & \cdots & \cdots & \cdots \\ X_MX_1 & X_MX_2 & \cdots & X_M^2 \end{Bmatrix}\begin{Bmatrix} Y_1^2 & Y_1Y_2 & \cdots & Y_1Y_N \\ Y_2Y_1 & Y_2^2 & \cdots & Y_2Y_N \\ \cdots & \cdots & \cdots & \cdots \\ Y_NY_1 & Y_NY_2 & \cdots & Y_N^2 \end{Bmatrix}\begin{Bmatrix} q_{11} \\ q_{12} \\ \cdots \\ q_{MN} \end{Bmatrix}\mathrm{d}x\mathrm{d}y$$

$$-\frac{\omega^2}{2}\int_0^a\int_0^b\rho h\begin{Bmatrix} X_1^2 & X_1X_2 & \cdots & X_1X_M \\ X_2X_1 & X_2^2 & \cdots & X_2X_M \\ \cdots & \cdots & \cdots & \cdots \\ X_MX_1 & X_MX_2 & \cdots & X_M^2 \end{Bmatrix}\begin{Bmatrix} Y_1^2 & Y_1Y_2 & \cdots & Y_1Y_N \\ Y_2Y_1 & Y_2^2 & \cdots & Y_2Y_N \\ \cdots & \cdots & \cdots & \cdots \\ Y_NY_1 & Y_NY_2 & \cdots & Y_N^2 \end{Bmatrix}\begin{Bmatrix} q_{11} \\ q_{12} \\ \cdots \\ q_{MN} \end{Bmatrix}\mathrm{d}x\mathrm{d}y=0. \quad (14)$$

The vibration equation of the composite sheet can be obtained with Eq. (15):

$$(\boldsymbol{K}-\omega^2\boldsymbol{M})\boldsymbol{q}=0. \quad (15)$$

3 Validation of laminated composite material model

The materials used in the test are carbon fiber-reinforced ceramic matrix composites, which are composed of carbon fibers and ceramic matrixes. The impregnation method is used as the method for material preparation. Carbon fiber is woven into a desired shape, impregnated with a ceramic material, and then fired to form the desired composite material. The material produced does not actually have an interface, but in the modeling process, it needs to be equivalent to a laminated structure. The model herein ignores the existence of an interface and assumes that layers are perfectly bonded together. A simplified diagram of the composite preparation process is shown in Fig. 1. The composite material property values provided by the manufacturer are shown in Table 1.

In this work, modal hammering and frequency sweeping tests are performed on a composite material sheet. The fourth-order natural frequency of the composite sheet was measured through experiments. Then, the composite material is modeled theoretically, and its each order natural frequency is calculated. The errors of the theoretical calculation results and experimental test results are compared to determine the accuracy of the theoretical model.

The modal test is used to obtain the natural frequency of the composite sheet. The system consists of two parts: A modal hammer test system and a sweep test system. The test sample is a plate composite (orthogonal laminate). The modal test chart of the composite material is shown in

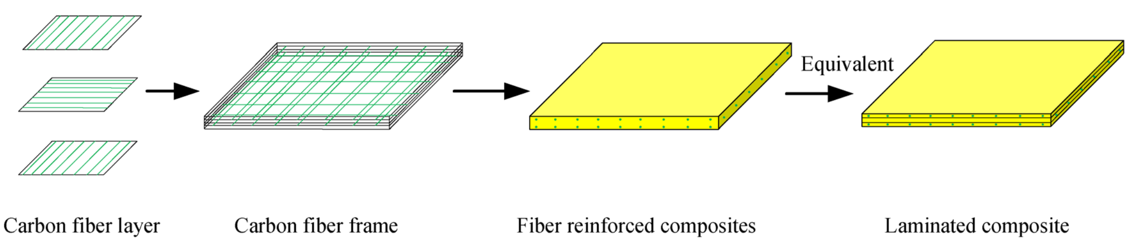


Fig. 1 Production process of fiber-reinforced composites.

Table 1 Composite structural parameter values and material parameter values

E_1	E_2	ν_{12}	G_{12}	ρ	Layer angle
33.5 GPa	4.5 GPa	0.3	5 GPa	1600 kg/m^3	0°/90°/0°

Note: E_1, the elastic modulus parallel to the fiber direction; E_2, the elastic modulus perpendicular to the fiber direction; G_{12}, the shear elastic modulu; ν_{12}, Poisson's ratio; ρ, the average density of the composite material.

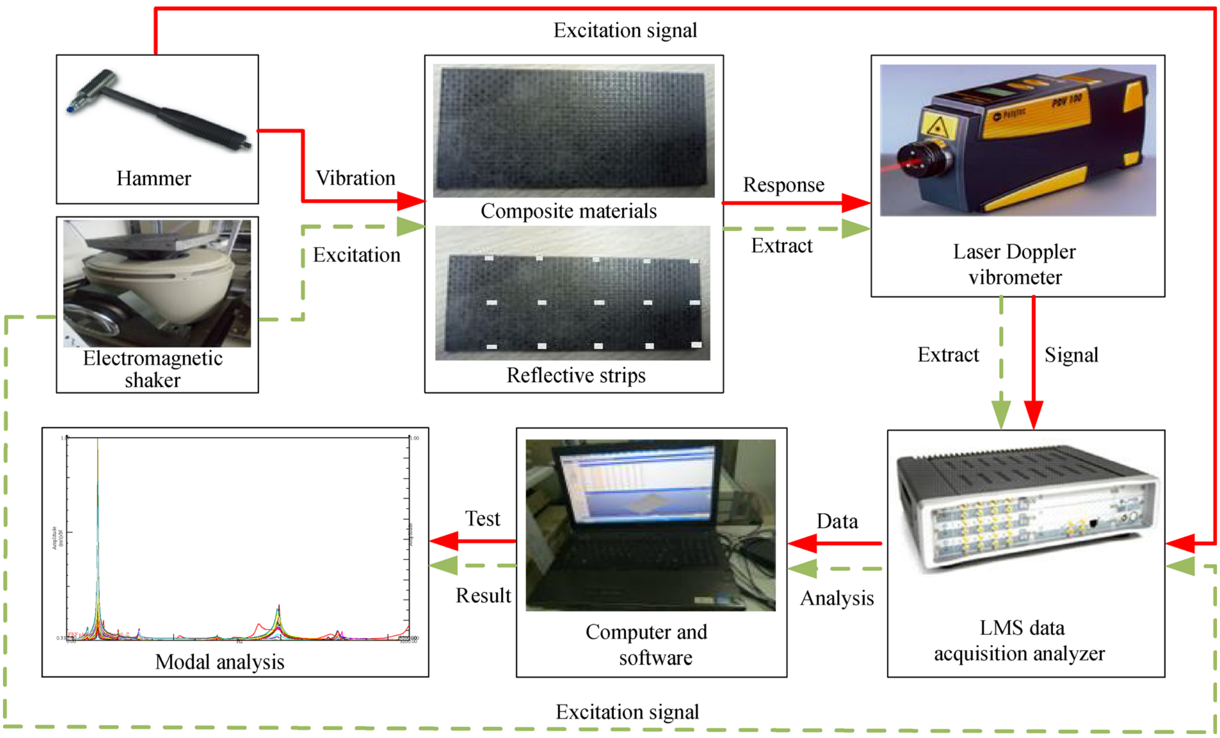


Fig. 2 Modal test chart of composite material.

Fig. 2, in which the solid line indicates the hammer test and the dotted line indicates the frequency sweep test. The techniques split into two parts are the same, except for the excitation modes.

The test sample is an orthogonally laminated structure with an overall size of 100 mm × 50 mm × 3 mm and average density of 1600 kg/m^3. The points to be tested are determined on the upper surface of the object to be tested, and reflective strips are posted on the points to be measured to reflect the laser signal. The distribution of the reflective strips is shown in Fig. 3.

First, the sample is mounted on the experiment rig with a clamping device, the constraint of which is 20 mm along the length direction; the other side is free. The length and thickness of the sample are along the x- and z-axis, respectively.

Second, the response signal is reflected by the reflective stickers that are attached on the free end to obtain the response signal by the laser Doppler vibrometer. The excitation signal and the response signal are processed and analyzed by the LMS data acquisition instrument, and the distribution range of the natural frequencies and mode shapes are obtained.

Finally, the sample is fixed on a vibration table, and the basic frequency sweep excitation is performed. The response signal is obtained by the reflective sticker on the free side. The precise natural frequencies are obtained in the sweep test.

The four mode shapes and their natural frequencies measured in the test are selected and compared with the mode shapes and natural frequencies calculated using the analytical model. The comparison results are shown in Table 2. The theoretically calculated natural frequency error does not exceed 2.5% relative to the measured natural frequency. This result indicates that the analytical model of the laminated composite material established in this work is reliable. Therefore, this model can be used to analyze the influence of structural parameters on the natural frequencies of composite materials.

4 Analysis of influencing factors of natural frequencies of laminated composite materials

The natural frequencies of fiber-reinforced laminated

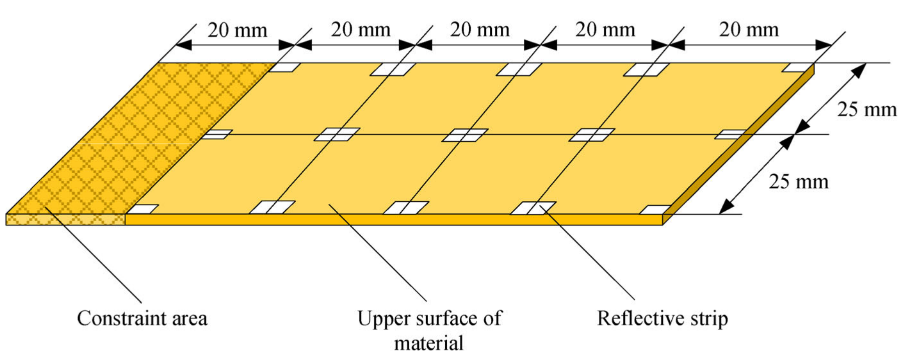
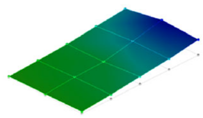
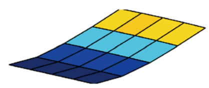
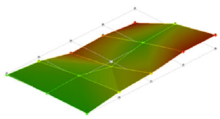
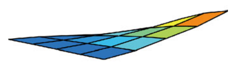
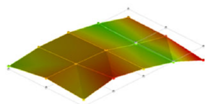
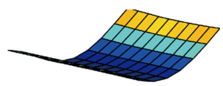
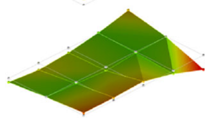
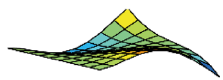


Fig. 3 Simplified diagram of the distribution of laser reflection strips.

Table 2 Comparison of mode shapes and natural frequencies from theoretical calculation and experimental test

i	Test frequency, ω_i^*/Hz	Test mode shape	Theoretical frequency, ω_i/Hz	Theory mode shape	Error, $\frac{\omega_i^*-\omega_i}{\omega_i^*}$/%
1	292.195		298.457		–2.140
2	810.355		792.698		2.179
3	1874.250		1894.750		–1.094
4	2531.530		2548.850		–0.684

composites is closely related to structural parameters, such as the number of layers, thickness, and layer angle. In engineering, clear requirements are established for the natural frequencies of fiber-reinforced composite materials. To meet the needs of the project and on the basis of the analytical model of composite materials herein, this study systematically analyzes the influence of the structural parameters of composite materials on natural frequency. Material density and each material parameter value are provided in Table 1. The material density and parameter value are provided in Table 1. The composite structure of the different layers, different thicknesses, and different angles is shown in Fig. 4, where h is the thickness of a single layer.

4.1 Effect of the number of layers on natural frequency

The study of the effect of the number of layers on the natural frequencies of composite materials is based on the premise that the thickness of the composite material is constant. In this work, the most common orthogonal lay-up structure (i.e., the fiber direction of each layer is 0° or 90° and the material properties and thicknesses are the same) is adopted to study the influence of the number of layers on the natural frequencies of composite materials. The fiber direction of the odd-numbered layers is 0°, and that of the even-numbered layers is 90°. The structural property of the material are shown in Table 3.

The graph drawn according to the obtained natural frequencies of the first four orders with different layers is shown in Fig. 5. Figure 5 reveals that the number of layers exerts a minimal effect on the natural frequencies of the first two orders and a large effect on the natural frequencies of the third and fourth orders; however, the result cannot clearly reflect the influence law. Therefore, the layered numbers are distinguished according to the odd and even layers. The variation curve graph of the natural frequency of the composite material with the number of layers is redrawn in Fig. 6.

Figure 6 shows that when the number of layers is an odd number, the first- and second-order natural frequencies decrease slowly, and the third-order natural frequency

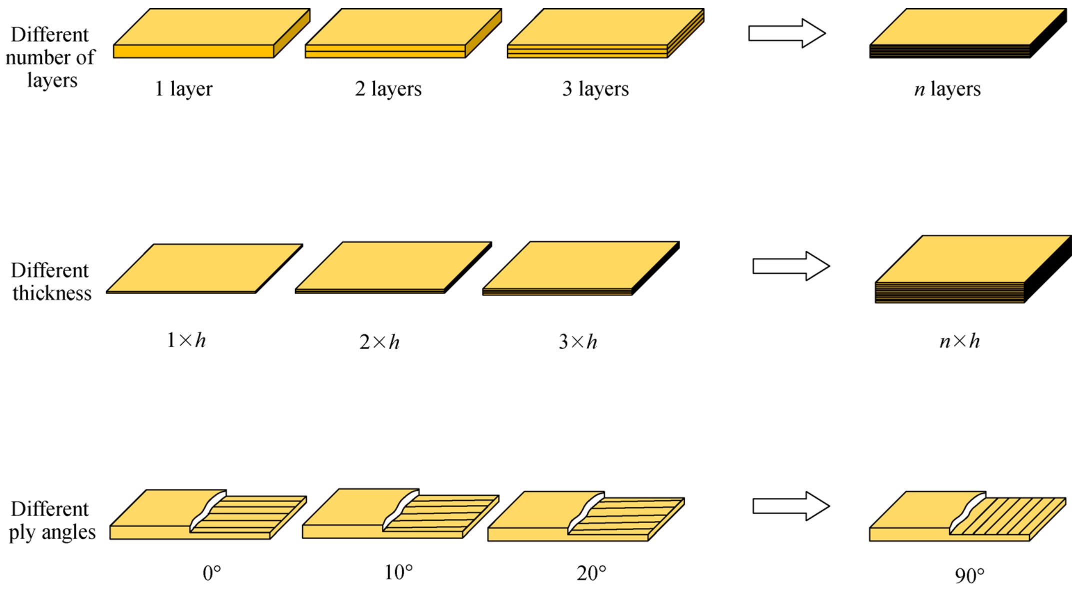


Fig. 4 Laminated composite materials with different structures.

Table 3 Structural parameters of composite materials with different layers

Length	Width	Thickness	Number of layers	Thickness of each layer
80 mm	50 mm	3 mm	n	$3/n$ mm

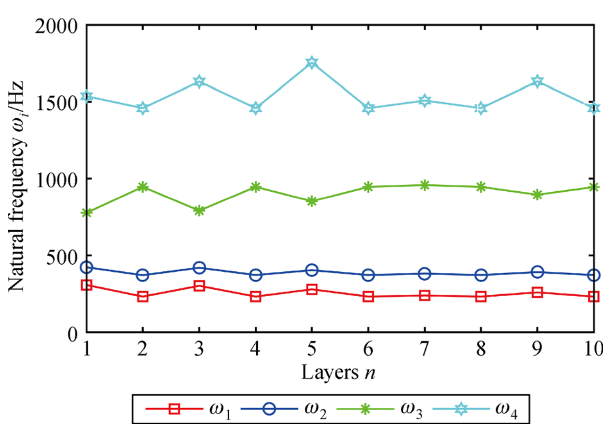


Fig. 5 Effect of the number of layers on the natural frequencies of composites.

increases as the number of layers increases. The fourth-order natural frequency of the composite material is most affected by the number of layers. When the number of layers is an even number, the natural frequency of the first four orders of the composite material changes slightly with the number of layers. Moreover, the natural frequency of the composite material can be considered to be relatively stable when the number of layers is an even number.

4.2 Effect of thickness on natural frequency

For laminated composite materials with an orthogonal ply structure and uniform layer thickness, their thickness is increased in this work by increasing the number of layers to study the effect of different thicknesses on the natural frequencies of composite materials. In particular, this study sets the material size parameters, which are shown in Table 4.

The graph drawn according to the obtained natural frequencies of the first four orders with different numbers of layers is shown in Fig. 7. As shown in Fig. 7, thickness exerts a large effect on the natural frequencies of the first four orders of the composite material. As the number of layers increases, the natural frequencies of the first four orders of the composite material increase greatly. In the comparison of the first four orders of natural frequencies of the composite material, we find that the growth rates of the first and second orders are lower than those of the others. The analysis results indicate the significant impact of composite material thickness on higher-order natural frequencies.

4.3 Effect of layer angle on natural frequency

4.3.1 Effect of layer angle on the natural frequencies of single-layer composites

In this work, laminated composites with a unidirectional layer angle are used as research objects. The fiber laying angle θ is changed to study the influence of the fiber laying angle of composite materials on their natural frequency. The selected size parameters of the composite material are shown in Table 5.

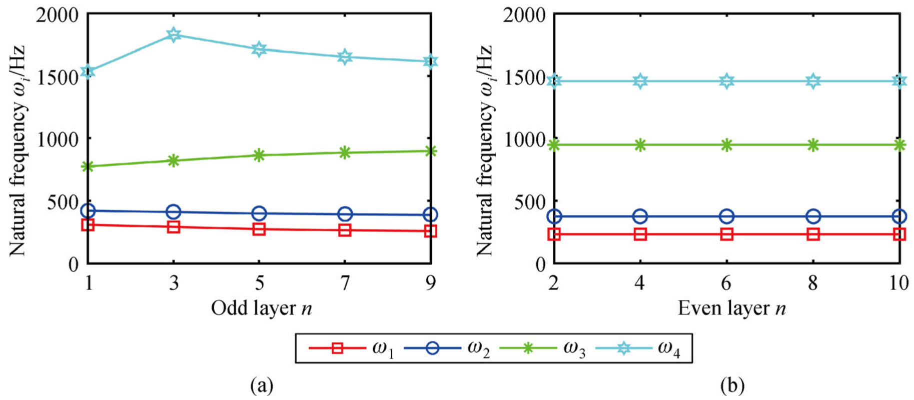


Fig. 6 Effect of odd and even numbers of layers on the natural frequencies of composite materials. (a) Odd-numbered layers; (b) even-numbered layers.

Table 4 Structural parameters of composite materials with different thicknesses

Length	Width	Thickness	Thickness of each layer	Layer angle
80 mm	50 mm	$1 \times n$ mm	1 mm	0°/90°/0°/90°/0°/90°/· · ·

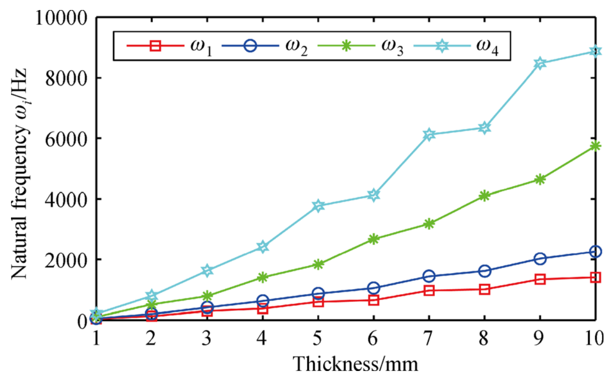


Fig. 7 Variation of natural frequencies of materials with thickness.

Table 5 Structural parameters of composite materials with different angles

Length	Width	Thickness	Layer	Layer angle
80 mm	50 mm	3 mm	1	θ°

The graph drawn according to the obtained natural frequencies of the first four orders with different layer angles is shown in Fig. 8. As shown in Fig. 8, the layer angle exerts a large effect on the first four natural frequencies of the composite material. When the layer angle gradually increases from 0° to 90°, the first-order natural frequency of the composite material gradually decreases. The second-order natural frequency increases with the increase of the layer angle from 0° to 30°, but it decreases gradually as the layer angle exceeds 40°. The third-order natural frequency increases as the layer angle increases from 0° to 50° and then begins to decrease. The

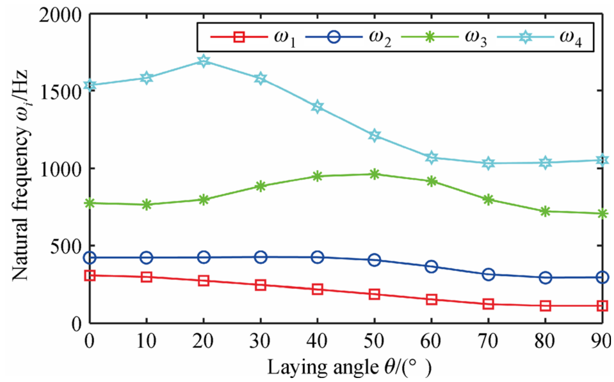


Fig. 8 Variation of natural frequencies of materials with layer angle.

fourth-order natural frequency increases as the layer angle increases from 0° to 30° and then decreased rapidly. When the layer angle is 60°, the reduction rate slows down and gradually stabilizes.

4.3.2 Effect of relative layer angles on the natural frequencies of two layers of composite material

A double-layer laminated composite material is taken as the research object. The relative layer angle θ between the two layers of the material is changed to study the influence of relative layer angles on the natural frequencies of composite materials. The size parameters of the composite material are shown in Table 6.

The graph drawn according to the obtained natural frequencies of the first four orders with different relative layer angles is shown in Fig. 9. As shown in Fig. 9, relative layer angles exert an obvious influence on natural frequencies. The first-order natural frequency gradually decreases as the relative layer angle increases from 0° to 90°. The second-order natural frequency increases as the relative layer angle increases from 0° to 30° and decreases gradually as the relative layer angle exceeds 40°. The third-order natural frequency increases as the relative layer angle

Table 6 Structural parameters of composite materials with different relative angles

Length	Width	Thickness	Layers	Thickness of each layer
80 mm	50 mm	3 mm	2	1.5 mm

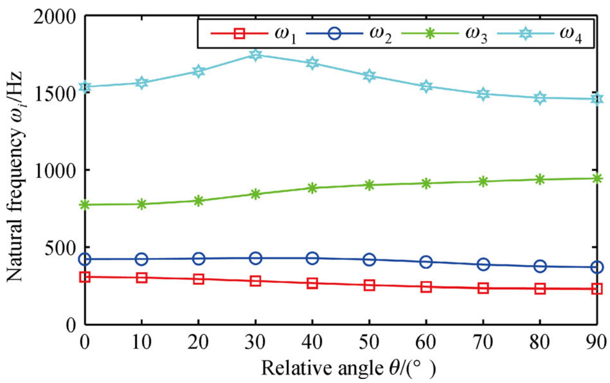


Fig. 9 Variation of natural frequencies of materials with relative layer angle.

increases. The fourth-order natural frequency increases as the relative layer angle increases from 0° to 30° and then gradually decreases.

In general, the effect of layer angles on the natural frequencies of the first and second orders is small; the first two orders of natural frequencies decrease slowly with the increase of the layer angle. By contrast, the effects of layer angles on the third and fourth orders of natural frequencies are large. With the increase of the layer angle, the third-order natural frequency increases slightly, and the fourth-order natural frequency increases first and then decreases. When the ply angle is 20°, 30°, or 50°, the high-order natural frequency of the composite material changes suddenly.

4.4 Analysis of coupling of influencing factors of natural frequencies of composite materials

Only a few laminated structures are commonly used in actual production and application. The difference in composite structures is affected by the coupling of the number of layers, fiber angle, and lay-up sequence. The composite laminate structure analyzed in this work includes symmetrical laminated structure 1 (represented by SY1), symmetrical laminated structure 2 (represented by SY2), orthogonal laminated structure (represented by OR), oblique laminated structure 1 (represented by OB1), oblique laminated structure 2 (represented by OB2) and quasi-isotropic laminated structure (represented by QI). The material parameter values are shown in Table 7. The fiber-reinforced composite materials with the above structures are defined as follows.

1) The symmetric laminated structure is described as follows: Symmetric laminated means that the material property, thickness, and fiber direction of each single layer are mirror-symmetrical with respect to the middle plane. Hence, SY1 in this work refers to the (0°/90°/90°/· · ·/90°/90°/0°) laying method, that is, the layer angle of the first layer and the last layer is 0°, and the layer angles of other layers are all 90°, and SY2 refers to the (0°/45°/45°/· · ·/45°/45°/0°) laminating method, that is, the layer angle of the first layer and the last layer is 0°, and the layer angles of other layers are all 45°.

2) The orthogonal laminated structure is described as follows: Layer thicknesses are the same, and the layer angles are 0° and 90° alternately. The lay-up form in this work is (0°/90°/0°90°/0°/· · ·).

3) The oblique laminated structure is described as follows: The layer thicknesses of the composite material are the same, and $+\theta$ and $-\theta$ are alternately laminated. Hence, OB1 used in this work refers to the (45°/–45°/45°/–45°/45°/· · ·) layering method, and OB2 refers to the (30°/–30°/30°/–30°/30°/· · ·) layering method.

4) The quasi-isotropic laminated structure is described as follows: The layer materials have the same thickness, the fiber direction of each layer is arranged in the order of $\theta_k = \pi(k-1)/n$, k is the kth layer, n is the total number of composite materials, and θ is the fiber lay-up angle.

Different composite material structures are formed by different layer angle and different numbers of layers. The natural frequencies of composite materials change with different structures. To obtain the change law of natural frequency, we need to analyze the influence of the above two factors on the natural frequencies of composite materials. Figure 10 presents laminated composite materials with different structures.

Table 7 Structural parameters of composite materials with different structures

Length	Width	Thickness	Layers	Thickness of each layer
80 mm	50 mm	3 mm	n	3/n mm

4.4.1 Effect of composite material structure on first-order natural frequency

Table 7 shows the structural parameters of composite materials, based on which the effects of the coupling of influencing factors on natural frequency are analyzed. Given the constraints of structure type, the number of layers ranges from 3 to 10, and the fiber layer angle and number of layers are bivariate. The 2D curve graph and 3D histogram of the first-order natural frequency varying with the number of layers are shown in Fig. 11.

As shown in Fig. 11, the first-order natural frequency of the symmetrical laminated structure gradually decreases as the number of layers increases. The tendency of the first-order natural frequency of the orthogonal laminated

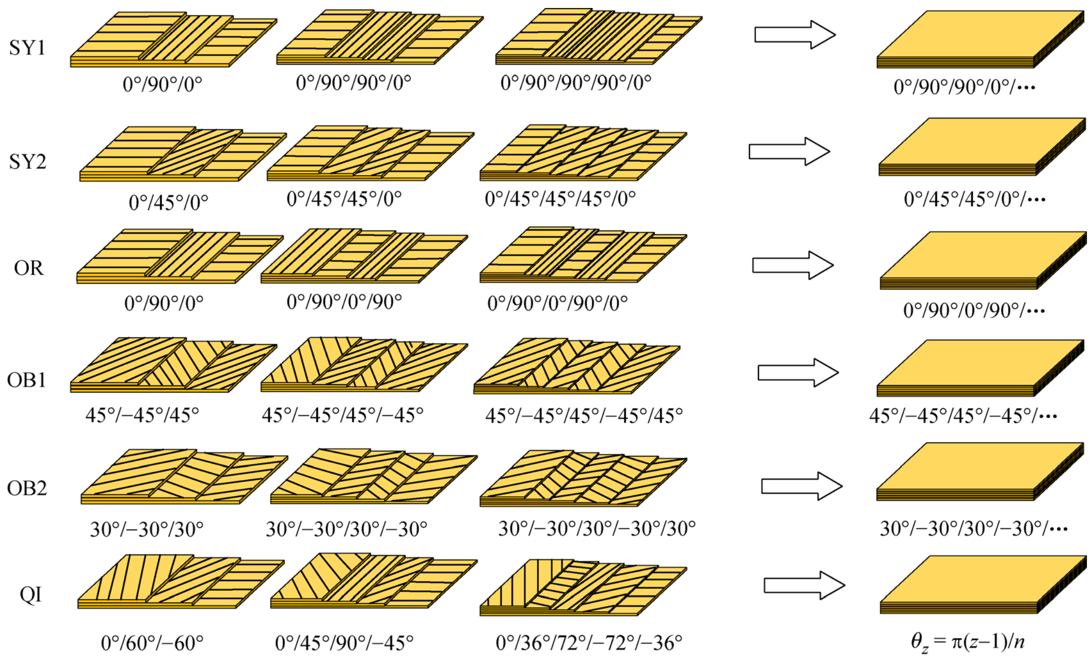


Fig. 10 Laminated composite materials with different structures.

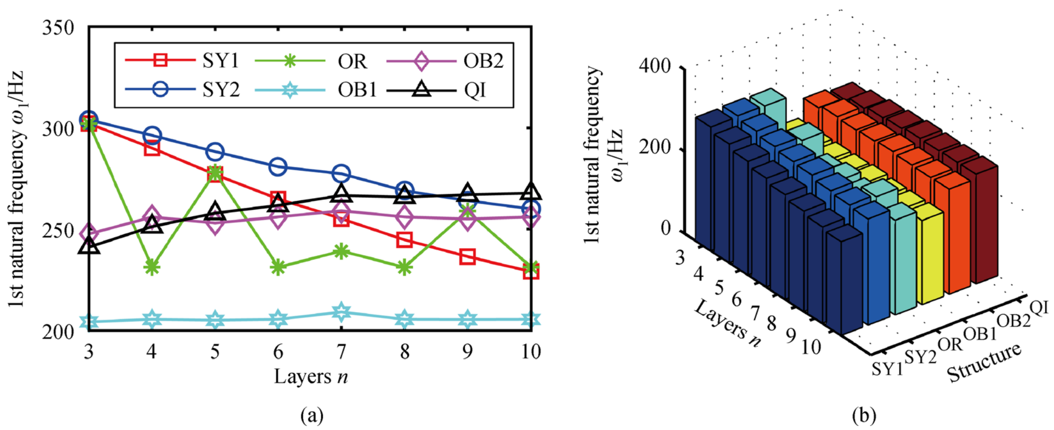


Fig. 11 Effects of different structures on the first-order natural frequency. (a) 2D curve graph; (b) 3D histogram.

structure remains unchanged when the number of layers is an even number, whereas it decreases first and then increases when the increased number of layers is an odd number. The tendency for the oblique ply composite appears relatively stable and less affected by the number of layers; meanwhile, that for the quasi-isotropic laminated structure gradually increases with the number of layers.

4.4.2 Effect of composite material structure on second-order natural frequency

The 2D curve graph and 3D histogram of the second-order natural frequency varying with the number of layers are shown in Fig. 12.

As shown in Fig. 12, the second-order natural frequency of the symmetrical laminated structure gradually decreases as the number of layers increases. The tendency of the second-order natural frequency of the orthogonal laminated structure remains unchanged when the number of layers is an even number, whereas it decreases first and then increases when the increased number of layers is an odd number. The tendency for the oblique ply structure is the same as that for the orthogonal laminated structure; meanwhile, that for the quasi-isotropic laminated structure increases first and then decreases with the increase in the number of layers.

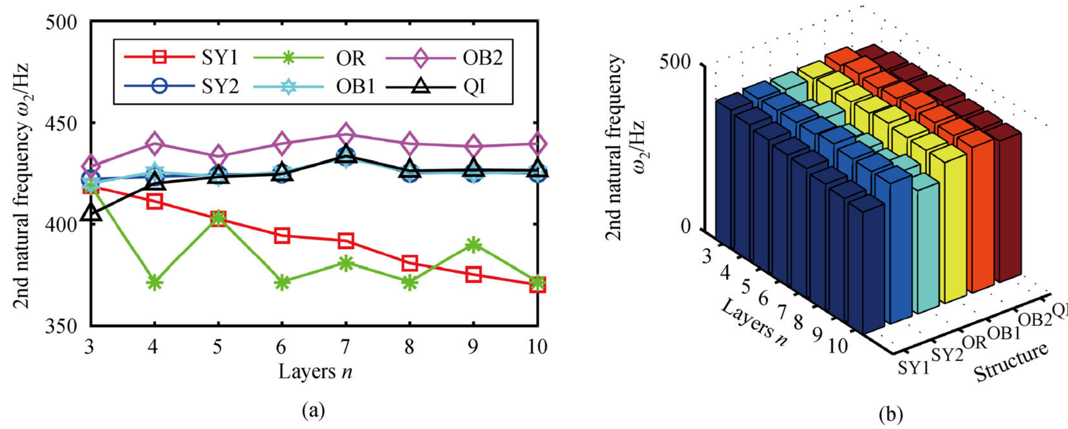


Fig. 12 Effects of different structures on the second-order natural frequency. (a) 2D curve graph; (b) 3D histogram.

4.4.3 Effect of composite material structure on third-order natural frequency

The 2D curve graph and 3D histogram of the third-order natural frequency varying with the number of layers are shown in Fig. 13.

As shown in Fig. 13, the third-order natural frequency of the symmetrical laminated structure gradually increases as the number of layers increases. The tendency of the third-order natural frequency of the orthogonal laminated structure remains unchanged when the number of layers is an even number, whereas it increases first and then decreases when the increased number of layers is an odd number. The tendency for the oblique ply structure is the same as that for the orthogonal laminated structure; meanwhile, that for the quasi-isotropic laminated structure decreases with the increase in the number of layers.

4.4.4 Effect of composite material structure on the fourth-order natural frequency

The 2D curve graph and 3D histogram of the fourth-order natural frequency varying with the number of layers are shown in Fig. 14.

As shown in Fig. 14, the fourth-order natural frequency of the symmetrical laminated structure increases first and then decreases as the number of layers increases. The tendency of the fourth-order natural frequency of the orthogonal laminated structure remains unchanged when the increased number of layers is an even number and becomes irregular when the number is an odd one. The oblique ply structure maintains the same fourth-order natural frequency when the number of layers is an even number, whereas it increases first and then decreases when the increased number of layers is an odd one. The fourth-order natural frequency of the quasi-isotropic laminated structure gradually increases with the increase in the number of layers.

The trends of the first four orders of natural frequency values of the composite materials with different structures affected by the number of layers are shown in Fig. 15. As shown in Figs. 15(a) and 15(b), the first- and second-order natural frequencies of the SY1 and SY2 composites are less affected by the number of layers, that is, they gradually decline as the number of layers increases. The third-order

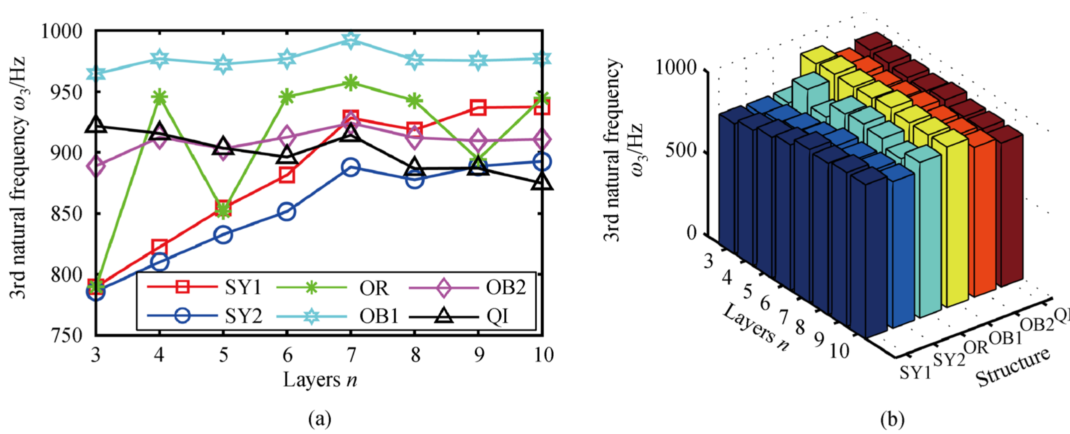
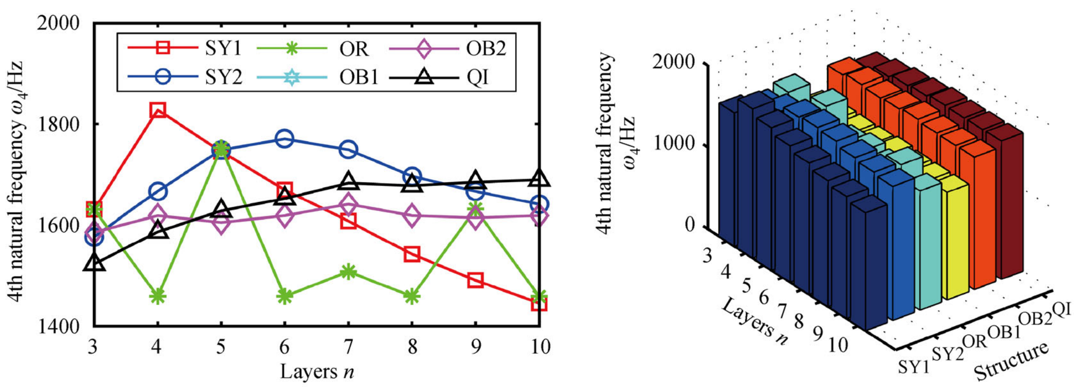
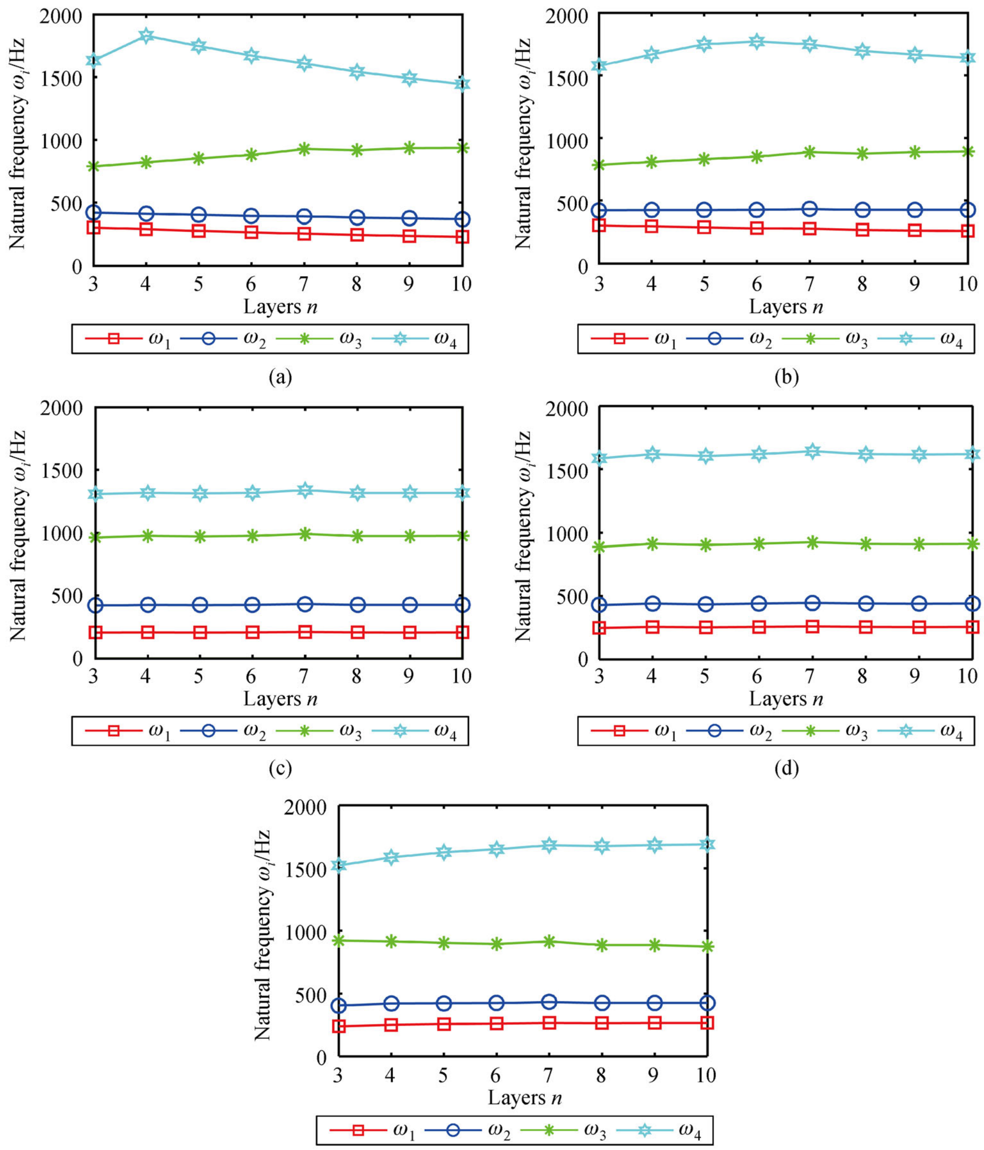


Fig. 13 Effects of different structures on the third-order natural frequency. (a) 2D curve graph; (b) 3D histogram.



(a) (b)

Fig. 14 Effects of different structures on the fourth-order natural frequency. (a) 2D curve graph; (b) 3D histogram.



(a) (b) (c) (d) (e)

Fig. 15 Effects of the number of layers on the natural frequencies of different structures. (a) SY1; (b) SY2; (c) OB1; (d) OB2; (e) QI.

natural frequency increases with the increase in the number of layers while the fourth-order natural frequency is greatly affected by the number of layers as it increases first and then decreases as the number of layers increases.

As shown in Figs. 15(c) and 15(d), the natural frequencies of the OB1 and OB2 composite materials are relatively stable. The natural frequencies of the first four orders are less affected by the number of layers. Figure 15(e) reflects the change law of the natural frequency of QI composites with the number of layers. The effect of the number of layers on the natural frequencies of the first four orders of the QI composite is relatively gentle, and only the fourth-order natural frequency increases slowly with the increase in the number of layers.

5 Conclusions

An analytical modeling method that comprehensively considers the influence of structural parameters on the natural frequency of composite materials was proposed, and the accuracy of the model was verified through experiments. The influence law of various structural parameters on the natural frequency of composite materials was determined through analysis. The conclusions are summarized as follows:

1) An analytical model of composite material dynamics was established, and the effects of the number of layers, thickness, layer angle, and the coupling of various parameters were considered in the model.

2) For composite materials with a certain total thickness, the natural frequencies are stable when the number of layers is an even number and are influenced slightly when the number of layers is an odd number. The fourth-order natural frequency changes abruptly when the number of layers is 3.

3) For composite materials with a certain layer thickness, the first- and second-order natural frequencies increase gradually, whereas the third- and fourth-order natural frequencies change drastically with the increase of thickness.

4) For single-layer fiber-laminated composite materials, the first- and second-order natural frequencies of the structure slowly decrease with the increase in layer angle. The third- and fourth-order natural frequencies peak at 50° and 20°, respectively, and then gradually decrease as the layer angle increases. For the relative layer angle, only the fourth-order natural frequency has a peak at 30°; the other natural frequencies change slightly.

5) Composite materials with different special structures were analyzed in this work, and the coupling effects of the number of layers and fiber angles on the natural frequency of composite materials were considered. The natural frequencies of the OB and QI structures are the most stable, the natural frequency of the OR structure has the strongest designability, and the low-order natural frequency of the SY structure is more stable than the high-order one.

This research provides a theoretical basis for the precise preparation of composite materials and the optimization of their structure and properties.

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References

1. Guang M, Qu Y. Vibration and Acoustics of Composite Structures. Beijing: National Defense Industry Press, 2017 (in Chinese)
2. Gibson R F. Principles of Composite Material Mechanics. 4th ed. Boca Raton: CRC Press, 2016
3. Newman G K, Shambaugh R L, Harwell J H. Method for forming a fibers/composite material having an anisotropic structure. US Patent 2007/0290397, 2007-12-20
4. Norman M A M, Zainuddin M A, Mahmud J. The effect of various fiber orientations and boundary conditions on natural frequencies of laminated composite beam. International Journal of Engineering & Technology, 2018, 7(3.11): 67–71
5. Imran M, Khan R, Badshah S. Finite element analysis to investigate the influence of delamination size, stacking sequence and boundary conditions on the vibration behavior of composite plate. Iranian Journal of Materials Science & Engineering, 2019, 16(1): 11–21
6. Taheri-Behrooz F, Pourahmadi E. A 3D RVE model with periodic boundary conditions to estimate mechanical properties of composites. Structural Engineering and Mechanics, 2019, 72(6): 713–722
7. Ghasemi A R, Taheri-Behrooz F, Farahani S M N, et al. Nonlinear free vibration of an Euler-Bernoulli composite beam undergoing finite strain subjected to different boundary conditions. Journal of Vibration and Control, 2016, 22(3): 799–811
8. Afsharmanesh B, Ghaheri A, Taheri-Behrooz F. Buckling and vibration of laminated composite circular plate on Winkler-type foundation. Steel and Composite Structures, 2014, 17(1): 1–19
9. Ghaheri A, Keshmiri A, Taheri-Behrooz F. Buckling and vibration of symmetrically laminated composite elliptical plates on an elastic foundation subjected to uniform in-plane force. Journal of Engineering Mechanics, 2014, 140(7): 04014049
10. Ananda Babu A, Vasudevan R. Vibration analysis of rotating delaminated non-uniform composite plates. Aerospace Science and Technology, 2017, 60: 172–182
11. Khalid H M, Yasin M Y, Khan A H. Free vibration analysis of multilayered arches using a layerwise theory. IOP Conference Series: Materials Science and Engineering. International Conference on Mechanical, Materials and Renewable Energy, 2018, 377: 012168
12. Roque C M C, Martins P. Maximization of fundamental frequency of layered composites using differential evolution optimization. Composite Structures, 2018, 183: 77–83
13. Mukhopadhyay T, Naskar S, Karsh P K, et al. Effect of delamination

on the stochastic natural frequencies of composite laminates. Composites. Part B, Engineering, 2018, 154: 242–256
14. Zhao J, Choe K, Shuai C, et al. Free vibration analysis of laminated composite elliptic cylinders with general boundary conditions. Composites. Part B, Engineering, 2019, 158: 55–66
15. Xue Z C, Li Q H, Wang J F, et al. Vibration analysis of fiber reinforced composite laminated plates with arbitrary boundary conditions. Key Engineering Materials, 2019, 818: 104–112
16. Singh B N, Yadav D, Iyengar N G R. Natural frequencies of composite plates with random material properties using higher-order shear deformation theory. International Journal of Mechanical Sciences, 2001, 43(10): 2193–2214
17. Leissa A W, Martin A F. Vibration and buckling of rectangular composite plates with variable fiber spacing. Composite Structures, 1990, 14(4): 339–357
18. Vigneshwaran K, Rajeshkumar G. Effect of matrix material on the free vibration characteristics of Phoenix sp. fiber reinforced polymer matrix composites. Materials Today: Proceedings, 2018, 5(5): 11227–11232
19. Cevik M. Effects of fiber orientation on out-of-plane and in-plane natural frequencies of angle-ply laminated composite arches. Journal of Reinforced Plastics and Composites, 2009, 28(1): 59–71
20. Zhong B, Li C, Li P. Modeling and vibration analysis of sectional-laminated cylindrical thin shells with arbitrary boundary conditions. Applied Acoustics, 2020, 162: 107184
21. Donadon B F, Mascia N T, Vilela R, et al. Experimental investigation of glued-laminated timber beams with Vectran-FRP reinforcement. Engineering Structures, 2020, 202: 109818
22. Honda S, Narita Y. Natural frequencies and vibration modes of laminated composite plates reinforced with arbitrary curvilinear fiber shape paths. Journal of Sound and Vibration, 2012, 331(1): 180–191
23. Narita Y. Layerwise optimization for the maximum fundamental frequency of laminated composite plates. Journal of Sound and Vibration, 2003, 263(5): 1005–1016
24. Dey S, Mukhopadhyay T, Sahu S K, et al. Effect of cutout on stochastic natural frequency of composite curved panels. Composites. Part B, Engineering, 2016, 105: 188–202
25. Djordjević Z, Jovanović S, Stanojević M, et al. Optimization of fiber orientation angle of a hybrid Al/composite cardan shaft. Acta Technica Corviniensis—Bulletin of Engineering, 2018, 11(2): 99–101
26. Li C, Zhong B, Shen Z, et al. Modeling and nonlinear vibration characteristics analysis of symmetrically 3-layer composite thin circular cylindrical shells with arbitrary boundary conditions. Thin-Walled Structures, 2019, 142: 311–321
27. Fallahi H, Taheri-Behrooz F, Asadi A. Nonlinear mechanical response of polymer matrix composites: A review. Polymer Reviews, 2020, 60(1): 42–85
28. Wang B, Sun W, Xu K, et al. The nonlinear stability prediction and FEM modeling of high-speed spindle system with joints dynamic characteristics. Shock and Vibration, 2014, 2014: 1–12
29. Hyer M W, White S R. Stress Analysis of Fiber-Reinforced Composite Materials. Lancaster: DEStech Publications, Inc., 2009
30. Reddy J N. Mechanics of Laminated Composite Plates and Shells: Theory and Analysis. Boca Raton: CRC Press, 2003