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# Developing a new model for simultaneous scheduling of two grand projects based on game theory and solving the model with Benders decomposition

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**Abstract** Grand infrastructure projects, such as dam, power plant, petroleum, and gas industry projects, have several contractors working on them in several independent sub-projects. The concern of reducing the duration of these projects is one of the important issues among various aspects; thus, our aim is to fulfill the requirements by using the game theory approach. In this study, a mixed-integer programming model consisting of game theory and project scheduling is developed to reduce the duration of projects with a minimum increase in costs. In this model, two contractors in successive periods are entered into a step-by-step competition by the employer during dynamic games, considering an exchange in their limited resources. The optimum solution of the game in each stage are selected as the strategy, and the resources during the game are considered to be renewable and limited. The strategy of each contractor can be described as follows: 1) share their resources with the other contractor and 2) not share the resources with the other contractor. This model can act dynamically in all circumstances during project implementation. If a player chooses a non-optimum strategy, then this strategy can immediately update itself at the succeeding time period. The proposed model is solved using the exact Benders decomposition method, which is coded in GAMS software. The results suggest the implementation of four step-by-step games between the contractors. Then, the results of our model are compared with those of the conventional models. The projects' duration in our model is reduced by 22.2%. The nominal revenue of both contractors has also reached a significant

value of 46078 units compared with the relative value of zero units in the original model. Moreover, we observed in both projects the decreases of 19.5%, 20.9%, and 19.7% in the total stagnation of resources of types 1, 2, and 3, respectively.

**Keywords** project scheduling, resource leveling between projects, constrained resources, game theory, Benders decomposition

## 1 Introduction

The increasing complexity of managing and scheduling activities, considering a set of limited resources, at the time of project implementation has led researchers become interested on how to use resources for reducing the time needed to complete projects. However, most studies in this area focus on resource leveling in project implementation, and several projects are implemented independently and simultaneously using the same resources. For example, in infrastructure projects, such as dams and power plants, and the petroleum industry, employers define several projects separately and assign them to independent contractors for implementation, in which each project is planned independently. With this approach, more renewable resources are required to increase the speed of project implementation. Consequently, some resources are not used full time and therefore can be shared to other projects, but a certain cost should be paid for such usage.

Few studies on project management and scheduling have addressed the problem of optimizing multi-project plans and maximizing the use of bedtimes of neighboring projects' resources. Thus, the problem examined in this study is on the allocation of resources in a bi-project situation. The key issues highlighted in this study are as follows.

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The maximum use of available resources in projects is limited. Thus, in scheduling projects in an independent structure, we require some particular resources to be used at certain times depending on the predecessor relationships between activities. By using this method, some renewable resources in their bedtime may be utilized.

On the one hand, by shortening the project duration, the payback period is reduced. This scenario causes the plan to become more attractive to investors and creates massive benefits for employers, given the time value of a completed plan. On the other hand, a shortened project duration also creates benefits for contractors owing to the reduction of fixed costs, and it frees up resources much earlier. In addition to the abovementioned advantages, employers can allow contractors to generally gain benefits from the project according to goals by giving special revenues to them.

In this study, we propose a mathematical model consisting of project scheduling and game theory, with the aim of maximizing the revenue of contractors. We can guide the two contractors through a framework, in which limited resources are exchanged and shared. Moreover, the contractors can reduce the time to complete the project with minimum additional costs and maximum revenues. The contributions of this study can be summarized as follows:

- Resource sharing is provided between two contractors, in consideration of the scheduling aspect, to maximize revenue;
- The projects' duration and resource stagnation time are utilized by using a new formulation;
- Game theory is used to formulate the abovementioned objectives;
- An exact solution method is applied on the basis of the Benders decomposition approach;
- Comprehensive computations are presented to verify the model.

The grand project in our study is a set of large-scale civil projects, such as dam, power plant, petroleum, and gas industry projects, with several contractors working concurrently on several independent sub-projects. Technical-economic feasibility studies must be justified before running the civil project plan. These sub-projects need several construction instruments and should be completed within a limited period and budget. An employer is a person (real or legal) who decides to execute a civil construction project, such as constructing a building, town, hospital, training complex, or any other civil works, with sufficient financial resources. The employer or investor may not be familiar with any civil issues.

## 2 Literature review

The field of project scheduling includes numerous different topics and various techniques. Thus, our aim is to

categorize the related literature into nine sub-sections for determining the literature gap.

### 2.1 Models to maximize the net present value in project scheduling

The idea of maximizing the net present value in projects was first proposed by Russell (1986). In addition, Etgar (1999) showed that resources beyond the time limit can significantly affect project duration, and a branch and bound approach for a model to realize project duration was investigated. Icmeli and Erenguc (1994) investigated models wherein adding resource constraints transforms a problem into a non-deterministic polynomial-time hard (NP-hard) model.

### 2.2 Project management and resource leveling

Neumann and Zimmermann (2000) proposed a heuristic polynomial method for different types of resource-leveling problems (RLPs) with minimum and maximum delay times between project activities. Browning and Yassine (2010) reviewed the literature on the scheduling of a multi-project plan with static resource constraints. Beşikci et al. (2015) proposed a multi-project problem (single contractor), including several projects allocated on the basis of the time of activities in which alternative resources are used. Ghoddousi et al. (2013) simultaneously considered three problems, particularly, a multi-mode resource-constrained project scheduling problem, a discrete time–cost tradeoff problem, and RLP. Naber and Kolisch (2014) investigated the problem of resource constraint for scheduling a project with flexible resources when resource utilization is not constant in an activity but instead can be adjusted from one period to another. Perez-Gonzalez and Framinan (2014) reviewed the literature on project control in multi-criteria scheduling problems by considering two or more categories of activities and by providing a single framework with a common definition. Takano et al. (2017) introduced a multi-period resource allocation algorithm to forecast project costs in a sequential competitive bidding state. A resource allocation formulation was developed as a mixed-integer linear programming model by using the piecewise linear estimation of the expected profit. Wang et al. (2017b) proposed a resource-constrained multi-project scheduling model in deterministic decision space. The duration of an activity was assumed to be an uncertain parameter, and two new robust indices were introduced to examine the performance of priority procedures under a random problem. Shariatmadari et al. (2017) proposed an integrated resource management procedure for concurrent project selection and scheduling while resource usage was optimized. A mixed-integer formulation was proposed for the model. In solving the model, a heuristic method was introduced to determine the feasible solutions.

### 2.3 Incorporation of game theory into project management and scheduling

Barough et al. (2012) proposed two game theory models based on two common conflicts in construction projects with the aim of finding the best output. Briand and Billaut (2011) considered the scheduling of a project, assuming that the network of activities is distributed among a set of actors. They modeled the duration of activities in a manner that each player is allowed to shorten the duration of some activities by spending more money. Avni and Tamir (2016) designed a game framework for the allocation of  $K$  machines to  $M$  jobs according to the load imposed by a job on a machine, the cost of activation of machines, and the possibility for a job to use the machine. Wang et al. (2017a) presented a theory of knowledge for collaboration. The method's factors, which affect knowledge collaboration, further affecting at least two members, were studied.

### 2.4 Using the Benders solution method for project scheduling problems

Chu and You (2013) integrated scheduling and dynamic optimization into complex batch processes with a general network structure. They developed an appropriate and efficient model on generalized Benders decomposition as a means to reduce computational complexity. Li (2015) employed a general class of allocation-type resource-constrained project scheduling problems by using the mixed Benders decomposition method for a staffing project.

### 2.5 Different objectives to reschedule projects

Rescheduling is required of a project which had been previously running. However, due to unpredicted actions, the earlier version of a scheduled program may no longer be applicable. In contrast to proactive programming that forecasts troubles by creating robust schedules, a case could be that a problem has occurred and a new program needs to be computed. This problem is named a rescheduling program. Calhoun et al. (2002) introduced a model to optimize the perturbation of an initial program by minimizing the number of activities, in which different starting times are obtained by the new program. Van de Vonder et al. (2007) presented an approach to reschedule the remaining events, in which the total amount of deviations of the new finishing dates from the initial model is optimized. This approach is designed according to the just-in-time theory, as introduced by Vanhoucke et al. (2001).

### 2.6 Networks for multi-project scheduling

In the real world, many dependent projects usually need to be scheduled concurrently. This task is essential for one,

two, or more projects that need to be handled in a parallel situation in accordance with at least one resource. Herroelen (2005) reported the significance of models for scheduling multiple projects. Pritsker et al. (1969) reported that one of the current ways to handle multiple projects is to design network activities into a "super-network" by adding a "super-source" and a "super-sink", in which a pool of resources is assumed. Confessore et al. (2007) assumed a set of multiple projects in which each project has its own resource, while an extra resource can be shared among projects. The benefit of aggregating various projects into a single network is the consideration of a proper foundation for the use of scheduling approaches for single projects besides the approach of multiple projects.

### 2.7 Scheduling and selecting projects

Chen and Askin (2009) introduced a multi-project model that includes two different decisions. First, a set of potential projects is assumed, and the projects to be conducted are determined. Second, the selected projects are planned according to common precedence and renewable resource limitations. Project selection and scheduling decisions are determined concurrently by considering the maximum net present value of the projects. Kolisch and Meyer (2006) aggregated project selection and scheduling into a single model for pharmaceutical research plans. The problem contained time-dependent resource requirements, and an objective was considered according to the net present value.

### 2.8 Resource request varying with time and project scheduling

The events in a classical project's scheduling problem need fixed volumes of renewable resources, in which the demand for resource at each period does not fluctuate within the duration of an event. Such a problem may be generalized by resource requirements that can be changed during the time. Hartmann (1999) developed a real-world medical study project with time-dependent resource requirements. In using this approach, each activity need is determined by laboratory tools during the last period of its duration. Cavalcante et al. (2001) studied a similar problem modeling and interpreted activities, in consideration of time-dependent resource needs for one renewable resource, as labor. Drezet and Billaut (2008) formulated time-dependent requirements for resources used by software developers. A set of minimum and maximum resource need for each period is computed.

### 2.9 Project scheduling in consideration of release dates and deadlines

Release time is the earliest date that an activity can be started, while the deadline shows the latest time that an

event must be finalized (Brucker et al., 1999). Previous formulations that consider release dates and deadlines were introduced by Klein and Scholl (1998; 2000), Kis (2005; 2006), and Drezet and Billaut (2008). Baptiste et al. (1999) presented a model to determine the cumulative scheduling model. Their approach offered a new case of project scheduling. In particular, unique renewable resources can be considered even without precedence relations. By using this method, release dates and deadlines for the activities can be considered, and the goal is to determine a feasible scheduling program.

### 2.10 Research gap

The aim of our research is to describe exactly its main contributions. The literature survey indicates that the studies associated with the applications of game theory in project scheduling are few. The existing works in this area are focused on certain minor aspects of game theory, such as the provision of contracts to employers, contractors, and sub-contractors. The full implementation of the game theory framework has not been studied, hence, our aim is to fulfill this gap. In this study, we decide to provide resource sharing between employers according to the game theory framework. Our method implies that resources cannot be used for varying reasons, such as predecessor and successor relationships at each time; thus, stagnation durations of constrained resources may exist. As a result, the activities of other projects can be conducted during the stagnation time in our method.

An important point is that contractors may prevent the exchange of their resources at certain periods and prioritize to their own projects; thus, mixed scheduling cannot be performed in practice. We attempt to overcome this issue by employing game theory and enter the parties involved in the plan, particularly the project contractors, into a cooperative game by allocating side revenues with the help of an employer. With the sharing of resources, some revenues can be gained, which may be due to: 1) the value of resources rented to the other contractor, 2) the time value of freed-up resources compared with those in conventional schedules for project completion, 3) the fixed costs removed from a project that is completed early, and 4) the shared benefits among contractors arising from the reduced payback period by the employer. A new mathematical model is proposed for the first time in this research based on the sharing of resources among contractors within a project schedule, and game theory is utilized to convince the contractors to share their resources. Finally, the Benders decomposition solution method is used to solve the proposed model. The proposed model for the scheduling problem in this study is based on the assumptions of a grand project, which include processors, resources, and type of cooperation. Thus, we use the terms

“grand project” and “contractors” to synchronize the problem description and the model for the readers.

## 3 Development of the mathematical model

The mathematical modeling of two projects is performed in a dynamic successive game, with the possibility of the resources being shared to maximize the players' (contractors') revenue. By exchanging the resources among contractors, the duration of both projects and the time of stagnation of resources in the projects can be minimized. The game between two contractors in this model occurs when a contractor is supposed to lend his resources to the other contractor. Once a contractor has lent his resource to the other contractor, the resource will be controlled by the latter contractor for a certain time as specified by the model. Upon the return of the resource to the owner-contractor, the game has not yet finished, as each contractor is assumed to have an authority to give or not to give a resource only for the resources owned by him. Therefore, at the time specified by the model to return the resource, this resource will be automatically returned to the resource owner. Each player can only choose from the following two strategies:

- 1) To give his resource in accordance with the schedule to the other player; or
- 2) Not to give his resource in accordance with the schedule to the other player, which then alters the scheduling and resource allocation of the project.

The model at this stage seeks to maximize the revenues of both players in the game, in which the employer enters to shorten the completion times of the two projects with a minimum increase in the costs. The most important aspect in this model is the simultaneous reduction of the resources' stagnation time and the projects' completion time while maximizing the players' revenue. The employers in this game can influence the specific strategies of the players by allowing them, within a specific ratio, to save on revenues based on the early completion of their respective project. The model can provide the best strategy for each player at different periods to maximize his revenue. If a player refuses to choose the optimum strategy, then the model automatically considers his constrained resources at the same time period for the other player, and the game moves to the next period accordingly.

### 3.1 Model assumptions

The amount of resources in the model is limited and renewable and has a constant value during the project. The resources are classified into three types, namely,  $r_1$ ,  $r_2$ , and  $r_3$ . The activities included in the projects are not breakable; the relationships between them are predecessor type; and

the activities in each project have no relationship with the activities in the other projects. The project scheduling is deterministic, and both projects simultaneously start at time zero. The constraint of the resources in the model and the game initialization are checked on the first day. The resource value and the handling costs are assumed to be constant in all periods, and the resources of both contractors are assumed to be the same and have an identical value. The game between two contractors occurs only when a contractor needs to lend his resource to the other contractor. In addition, the return of that resource to the owner-contractor occurs according to the model's scheduling in a deterministic pattern. Both projects are independently defined with the same employer. The model is presented in the succeeding sections.

### 3.2 Model formulation

#### Indices:

$i$  is the index of activity.

$j$  is used to capture the predecessor relationships between activities and entered into the program in constraint (1).

$r$  is the index of reversible resources.

$d$  denotes the period to perform activities and allocate resources to the activities, and it is considered daily.

$c$  is the index of the contractor/project (two contractors/projects are assumed in this study).

#### Parameters:

$J$  is the upper bound for the number of activities in a project.

$R$  is the upper bound for available resources.

$D$  is the upper bound for available days considered for scheduling.

$C$  is the upper bound for the number of contractors/projects.

$M$  is a large number for relaxing constraints.

$p(j, c)$  denotes the time required to perform activity  $j$  of project  $c$ .

$b(r, c)$  denotes the constrained renewable resource  $r$  in project  $c$  owned by the contractor.

$q(d)$  denotes a mediating parameter used to simplify the coding of the relationship between two continuous and discrete variables.

$l(r, j, c)$  is the quantity of resource  $r$  used in project  $c$  by activity  $j$  during its running period.

$relation(i, j, c)$  corresponds to the predecessor relationships between activities in the form of a table. A value of 1 indicates that in project  $c$  the predecessor of activity  $j$  is activity  $i$ ; otherwise, no relationship exists between the two activities in the project.

$s(r)$  is a value or cost (daily rent) for using each resource type  $r$  at each time period (according to the unit of money).

$h(r)$  is the cost paid by the contractor for using resource  $r$  of the other contractor (according to the unit of money).

$k(r)$  is the revenue the employer offers to the resource owner-contractor for each instance of the resource exchange between contractors. The revenue is applied to control the game when the project undergoes a delay due to a violation of the game optimum by a contractor (according to the unit of money).

$n(c, c')$  is the amount of money that the employer pays for each day of early completion of project  $c$  to contractor  $c'$  (according to the unit of money).

#### Model scalars:

$gg_1$  is the optimum time needed to complete project 1 in the first stage, and it is individually scheduled using the first contractor's resources.

$gg_2$  is the optimum time needed to complete project 2 in the first stage, and it is individually scheduled using the second contractor's resources.

The abovementioned values were obtained after solving the project scheduling model.

$\varepsilon$  is a very small number such as 0.001.

#### Variables:

$t(j, c)$  is the time or period to complete activity  $j$  in project  $c$ .

$x(j, d, c)$  is a binary (0 and 1) variable. The value is 1 if activity  $j$  of project  $c$  had been scheduled at time period  $d$ ; otherwise, 0.

$y(j, d, c)$  is an auxiliary variable used to activate 1 for constraints (5) or (7); otherwise, the variable  $y$  is 0.

$f$  is the value of the objective function, and it is equal to the summation of the maximum revenues of the two contractors in the game.

$g(c)$  is the auxiliary variable needed to linearize the completion time function of each project.

$z(d, r, c)$  is the difference between the amount of constrained resource  $r$  owned by contractor  $c$  and the amount of resource used by the same contractor at time period  $d$ . If the value is positive, then contractor  $c$  has used the resource of the other contractor at time period  $d$ . The value is equal to the quantity borrowed from the other contractor. If the value is negative, then contractor  $c$  has not used its entire resources, the resources are likely in their bedtime, or the resources have been borrowed by the other contractor.

$v(d, r, c)$  is the strategy of contractor  $c$  at time period  $d$  for each resource type  $r$  in the game. The value is 1 if the resource is shared; otherwise, 0.

$w(d, r, c)$  is the resource  $r$  that contractor  $c$  has borrowed from the other contractor at time period  $d$ .

**Objective function:**

$$f = \max \left\{ \begin{aligned} & n(1, 1) \times (gg_1 - g(1)) + n(1, 2) \times (gg_2 - g(2)) + \sum_{r=1}^R \sum_{d=0}^D s(r) \times w(d, r, 2) \\ & - \sum_{r=1}^R \sum_{d=0}^D h(r) \times w(d, r, 1) + \sum_{r=1}^R \sum_{d=0}^D k(r) \times v(d, r, 2) \end{aligned} \right\} \\ + \left\{ \begin{aligned} & n(2, 2) \times (gg_2 - g(2)) + n(2, 1) \times (gg_1 - g(1)) + \sum_{r=1}^R \sum_{d=0}^D s(r) \times w(d, r, 1) \\ & - \sum_{r=1}^R \sum_{d=0}^D h(r) \times w(d, r, 2) + \sum_{r=1}^R \sum_{d=0}^D k(r) \times v(d, r, 1) \end{aligned} \right\}. \quad (1)$$

**Constraints:**

$$t(j, c) - t(i, c) \geq p(j, c), \forall i, j \in \text{relation}(i, j, c), \quad (2)$$

$$\sum_{c=1}^C \sum_{j=0}^J l(r, j, c) \times x(j, d, c) \leq \sum_{c=1}^C b(r, c), \forall d, r, \quad (3)$$

$$q(d) \times x(j, d, c) \leq t(j, c), \forall j, d, c, \quad (4)$$

$$\begin{aligned} & -t(j, c) + M \times x(j, d, c) - M \times y(j, d, c) \\ & \geq \varepsilon - M - q(d), \forall j, d, c, \end{aligned} \quad (5)$$

$$\begin{aligned} & -t(j, c) + M \times x(j, d, c) \geq -M - p(j, c) - q(d), \\ & \forall j, d, c, \end{aligned} \quad (6)$$

$$\begin{aligned} & p(j, c) \times x(j, d, c) + q(d) \times x(j, d, c) \\ & + M \times y(j, d, c) + t(j, c) \geq p(j, c) + \varepsilon + q(d), \\ & \forall j, d, c, \end{aligned} \quad (7)$$

$$g(c) \geq t(i, c), \forall i, c, \quad (8)$$

$$\begin{aligned} & \sum_{j=0}^J l(r, j, c) \times x(j, d, c) - b(r, c) = z(d, r, c), \\ & \forall d, r, c, \end{aligned} \quad (9)$$

$$z(d, r, c) \leq M \times v(d, r, c), \forall d, r, c, \quad (10)$$

$$-z(d, r, c) + \varepsilon \leq M \times (1 - v(d, r, c)), \forall d, r, c, \quad (11)$$

$$w(d, r, c) \leq M \times v(d, r, c), \forall d, r, c, \quad (12)$$

$$w(d, r, c) \leq z(d, r, c) + M \times (1 - v(d, r, c)),$$

$$\forall d, r, c, \quad (13)$$

$$w(d, r, c) \geq z(d, r, c) - M \times (1 - v(d, r, c)),$$

$$\forall d, r, c, \quad (14)$$

where  $t(j, c)$ ,  $g(c)$ ,  $w(d, r, c) \geq 0$ , and  $x(j, d, c)$ ,  $y(j, d, c)$ ,  $v(d, r, c) \in \{0, 1\}$ .

Equation (1) corresponds to the objective function of the model and is composed of two parts. The first bracket refers to the first contractor's revenue, while the second bracket refers to the second contractor's revenue. The objective function of the model is to maximize the summation of both contractors' revenues. The revenues of both contractors are considered equal, which include the following four parts:

1) The revenue (positive) due to early completion of the project is compared with the conventional optimum schedule, which is without resource sharing. In addition to the above revenue, the contractor receives some revenue for early completion of the second project. The aim of this work is to make important both projects for both contractors, so both contractors attempt for early completion of both projects by these equations, respectively:

$$n(1, 1) \times (gg_1 - g(1)) + n(2, 2) \times (gg_2 - g(2)),$$

$$n(1, 2) \times (gg_2 - g(2)) + n(2, 1) \times (gg_1 - g(1)).$$

2) The revenue (positive) resulting from the lending of resource  $r$  to the other contractor is computed using:

$$\sum_{r=1}^R \sum_{d=0}^D s(r) \times w(d, r, 2) + \sum_{r=1}^R \sum_{d=0}^D s(r) \times w(d, r, 1).$$

3) The revenue (negative) resulting from the borrowing of resource  $r$  from the other contractor is computed using:

$$-\sum_{r=1}^R \sum_{d=0}^D h(r) \times w(d, r, 1) - \sum_{r=1}^R \sum_{d=0}^D h(r) \times w(d, r, 2).$$

4) A special revenue (positive) is used to perform a particular game considered by the employer. This revenue is important to avoid delays in the project, and it is calculated using:

$$\sum_{r=1}^R \sum_{d=0}^D k(r) \times v(d, r, 2) + \sum_{r=1}^R \sum_{d=0}^D k(r) \times v(d, r, 1).$$

Constraint (2) applies to the predecessor relationship between the two activities of  $i$  and  $j$ , as shown in the relation table in the succeeding section. Constraint (3) shows that the amount of resources in both projects at each time period does not exceed the resources of the two contractors in both projects ( $b(r, 1) + b(r, 2)$ ). This constraint allows for the common use of similar resources. Constraints (4)–(7) guarantee the relationships between constraints (2) and (3) for each project. Constraint (8) shows the completion time of both projects and the relationship of each completion time in each project. The completion time of a project is the time of which all activities of the project have been completed. Constraint (9) is a definition of  $z(d, r, c)$  and corresponds to the difference between the available resource  $r$  of each contractor and the amount of resources used by the same contractor at time period  $d$ . Constraints (10) and (11) are complementarily used to define and identify  $v(d, r, c)$ . Constraints (12)–(14) are complementarily used to define the essential variable of  $w(d, r, c)$ .

The maximum revenues of the parties (i.e., the optimum mixed model) are used to determine the resource exchange. In this model, instead of calculating separately the possible game scenarios at different periods, we obtain the best solution by computing the mixed model.

The resources are dynamically defined for all time periods. In this manner, we can determine the feasibility or infeasibility of a game according to the particular conditions of the project. Thus, if a contractor has not followed the optimum game strategy (i.e., the schedule designed for both projects by the employer) in one or more particular time periods, the value of  $v(d, r, c)$  can be regarded sufficiently related to the game between parties in that time period based on the progress of the projects. Therefore, the program is run again, and we update automatically the schedule plan in that period. This event is based on the progress of projects, aside from the previous schedule, and implemented only by considering the current state. By using this method, the allocation of resources to the activities can be determined. In addition, the event returns to the developed mixed model for the next periods and can provide the possibility of another round of resource sharing. Each player can also individually decide on sharing his resources or not. The worst-case scenario of

the model occurs when both players (contractors) avoid resource sharing in each period, indicating that both periods will return to the previous state. This worst case is equivalent to the optimum of the initial model.

## 4 Development of the Benders solution method

As the developed model is an NP-hard and a meta-heuristic method that does not provide an exact solution, we use the Benders decomposition solution method to obtain the exact optimum solution (Makui et al., 2016).

**Step 1:** Standardize the problem by minimizing the objective function and imposing the constraints in the model to be “larger than or equal” state.

**Step 2:** Drive the main problem (MP) from the model. Define initially the  $Z_{\text{lower}}$  and minimize it as follows:

$$\text{Min } Z_{\text{lower}}, \quad (15)$$

s.t.

$$\begin{aligned} Z_{\text{lower}} \geq & -\sum_{r=1}^R \sum_{d=0}^D k(r) \times v(d, r, 1) \\ & -\sum_{r=1}^R \sum_{d=0}^D k(r) \times v(d, r, 2), \end{aligned} \quad (16)$$

$$\begin{aligned} -\sum_{c=1}^C \sum_{j=0}^J l(r, j, c) \times x(j, d, c) \leq & -\sum_{c=1}^C b(r, c), \\ \forall d, r, \end{aligned} \quad (17)$$

$$\begin{aligned} & -\sum_{j=0}^J l(r, j, c) \times x(j, d, c) + b(r, c) \\ & \geq -M \times v(d, r, c), \forall d, r, c, \end{aligned} \quad (18)$$

$$\begin{aligned} & \sum_{j=0}^J l(r, j, c) \times x(j, d, c) - b(r, c) - \varepsilon \\ & \geq -M \times (1 - v(d, r, c)), \forall d, r, c. \end{aligned} \quad (19)$$

Integrate the calculated result to the MP that is composed of constraints with no continuous variables and contains integer variables only. Here, constraints (2), (9), and (10) are selected.

**Step 3:** Drive the sub-problem model.

We linearize the objective function by removing all integer variable forms such that only continuous variables remain as follows:

$$f = \min \left\{ n(1, 1) \times (g(1) - gg_1) + n(1, 2) \times (g(2) - gg_2) - \sum_{r=1}^R \sum_{d=0}^D s(r) \times w(d, r, 2) + \sum_{r=1}^R \sum_{d=0}^D h(r) \times w(d, r, 1) \right\} \\ + \left\{ n(2, 2) \times (g(2) - gg_2) + n(2, 1) \times (g(1) - gg_1) - \sum_{r=1}^R \sum_{d=0}^D s(r) \times w(d, r, 1) + \sum_{r=1}^R \sum_{d=0}^D h(r) \times w(d, r, 2) \right\}. \quad (20)$$

Then, we fix the integer variables and transfer them to the right part of the inequality to formulate the constraints of the sub-problem model. We also remove the constraints that do not contain any continuous variables.

$$t(j, c) - t(i, c) \geq p(j, c), \quad \forall i, j \in \text{relation}(i, j, c), \quad (21)$$

$$t(j, c) \geq q(d) \times \bar{x}(j, d, c), \quad \forall j, d, c, \quad (22)$$

$$-t(j, c) \geq -M \times \bar{x}(j, d, c) + M \times \bar{y}(j, d, c) \\ + \varepsilon - M - q(d), \quad \forall j, d, c, \quad (23)$$

$$-t(j, c) \geq -M \times \bar{x}(j, d, c) - M - p(j, c) - q(d), \\ \forall j, d, c, \quad (24)$$

$$t(j, c) \geq -p(j, c) \times \bar{x}(j, d, c) - q(d) \times \bar{x}(j, d, c) \\ - M \times \bar{y}(j, d, c) + p(j, c) + \varepsilon + q(d), \\ \forall j, d, c, \quad (25)$$

$$g(c) \geq t(i, c),$$

$$\forall i, c \approx \left\{ \begin{array}{l} g(1) - t(i, 1) \geq 0 \\ g(2) - t(i, 2) \geq 0 \end{array} \right\} \left\{ \begin{array}{l} \forall i \\ \forall i \end{array} \right\}, \quad (26)$$

$$-w(d, r, c) \geq -M \times \bar{v}(d, r, c), \quad \forall d, r, c, \quad (27)$$

$$-w(d, r, c) \geq -\sum_{j=0}^J l(r, j, c) \times \bar{x}(j, d, c) + b(r, c) \\ - M \times (1 - \bar{v}(d, r, c)), \quad \forall d, r, c, \quad (28)$$

$$w(d, r, c) \geq \sum_{j=0}^J l(r, j, c) \times \bar{x}(j, d, c) - b(r, c) \\ - M \times (1 - \bar{v}(d, r, c)), \quad \forall d, r, c. \quad (29)$$

With the abovementioned conditions, the linear sub-problem model can be obtained.

**Step 4:** Form the dual sub-problem (DSP).

We define a variable in the sub-problem for each constraint in the primal model. By using this method, we can define variable  $u_1$  of constraint (21), variables  $u_3$ – $u_7$  of constraints (22)–(26), and variables  $u_{11}$ – $u_{13}$  of constraints (27)–(29).

$$Z_{\text{DSP}} = \text{relation}(i, j, c) \times u_1(i, j) \times p(i, c) + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C u_3(i, c) \times \langle q(d) \times \bar{x}(j, d, c) \rangle \\ + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C u_4(i, c) \times \langle -M \times \bar{x}(j, d, c) + M \times \bar{y}(j, d, c) + \varepsilon - M - q(d) \rangle \\ + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C u_5(i, c) \times \langle -M \times \bar{x}(j, d, c) - M - p(i, c) - q(d) \rangle \\ + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C u_6(i, c) \times \langle -p(i, c) \times \bar{x}(j, d, c) - q(d) \times \bar{x}(j, d, c) - M \times \bar{y}(j, d, c) + p(i, c) + \varepsilon + q(d) \rangle \\ + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C u_7(i, c) \times \langle 0 \rangle + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C u_{11}(d, r, c) \times \langle -M \times \bar{v}(d, r, c) \rangle \\ + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C u_{12}(d, r, c) \times \langle -\sum_{j=0}^J l(r, j, c) \times \bar{x}(j, d, c) + b(r, c) - M \times (1 - \bar{v}(d, r, c)) \rangle \\ + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C u_{13}(d, r, c) \times \langle \sum_{j=0}^J l(r, j, c) \times \bar{x}(j, d, c) - b(r, c) - M \times (1 - \bar{v}(d, r, c)) \rangle, \quad (\text{DSP1})(30)$$



s.t.

$$u_7(i, 1) \leq n(1, 1) + n(2, 1), \quad (\text{DSP2})(31)$$

$$u_7(i, 2) \leq n(2, 2) + n(1, 2). \quad (\text{DSP3})(32)$$

### Steps of Benders decomposition

First, we address the MP model and initialize the integer variables to obtain the  $Z_{\text{lower}}$  value. Then, we substitute the values of the integer variables in the DSP model and obtain the optimum of the DSP, which is a corner point of the solution space of our dual model. By adding the values of

the optimum objective function to the MP and DSP models, we can achieve the upper bound and determine the value of the lower bound, which is equal to the value of  $Z_{\text{lower}}$ .

In the succeeding iteration, the feasible cut should be added to the MP model as a constraint as follows.

We set  $Z_{\text{lower}}$  to be larger than or equal to the sum of two values, namely, 1) the value of the initial objective function obtained for the integer variables and 2) the value of the DSP objective function obtained from the values of the dual variables in the previous step. This condition can be expressed as:

$$\begin{aligned} Z_{\text{lower}} \geq & - \sum_{r=1}^R \sum_{d=0}^D k(r) \times v(d, r, 1) - \sum_{r=1}^R \sum_{d=0}^D k(r) \times v(d, r, 2) \\ & + \text{relation}(i, j, c) \times \bar{u}_1(i, j) \times p(i, c) + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C \bar{u}_3(i, c) \times \langle q(d) \times x(j, d, c) \rangle \\ & + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C \bar{u}_4(i, c) \times \langle -M \times x(j, d, c) + M \times y(j, d, c) + \varepsilon - M - q(d) \rangle \\ & + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C \bar{u}_5(i, c) \times \langle -M \times x(j, d, c) - M - p(i, c) - q(d) \rangle \\ & + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C \bar{u}_6(i, c) \times \langle -p(i, c) \times x(j, d, c) - q(d) \times x(j, d, c) - M \times y(j, d, c) + p(i, c) + \varepsilon + q(d) \rangle \\ & + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C \bar{u}_7(i, c) \times \langle 0 \rangle + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C \bar{u}_{11}(d, r, c) \times \langle -M \times v(d, r, c) \rangle \\ & + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C \bar{u}_{12}(d, r, c) \times \langle - \sum_{j=0}^J l(r, j, c) \times x(j, d, c) + b(r, c) - M \times (1 - v(d, r, c)) \rangle \\ & + \sum_{j=0}^J \sum_{d=0}^D \sum_{c=1}^C \bar{u}_{13}(d, r, c) \times \langle \sum_{j=0}^J l(r, j, c) \times x(j, d, c) - b(r, c) - M \times (1 - v(d, r, c)) \rangle. \end{aligned} \quad (33)$$

After obtaining the exact values of the dual variables in the DSP model at each iteration, we need to add a feasible cut (obtained for the dual basic point in the iteration) to the MP model. The difference between the upper bound and the values on the right part represents the optimum cut.

These iterations are continued until the difference between the upper bound and lower bound becomes less than  $\varepsilon$  (i.e.,  $Z_{\text{lower}} - Z_{\text{upper}} \leq \varepsilon$ ), which is the termination condition of the Benders algorithm. The value of  $\varepsilon$  can be considered zero, where  $Z_{\text{lower}} = Z_{\text{upper}}$  represents the termination condition. If the termination condition is satisfied, then the model solution process will be stopped, and the optimum values will be obtained.

ships. The aim is to minimize the project completion time and maximize the revenue. A resource exchange between the two contractors is possible in this game as a means to maximize the contractors' revenues. Thus, we run the scheduling problem. In the succeeding step, the two contractors in the cooperative game decide on strategies of sharing or not sharing resources with the other. By comparing the results, we can investigate the model performance developed in the previous section. The comparisons include time-saving aspects and the revenues gained by the contractors and the efficiency of resource utilization. Both projects include activities labeled by numbers 1 to 7. We need the two activities of 0 and 8 to complete the activity network.

## 5 Numerical calculations

We present in this section the cooperation between the two contractors performing the activities of the two projects. The activities of each project have predecessor relation-

### 5.1 Tables and parameters: Typical sample

Table 1 shows the parameter  $p(j, c)$  corresponding to the time needed to implement activity  $j$  of project  $c$ . The upper

**Table 1** Time needed to implement activity  $j$  of project  $c$  ( $p(j, c)$  in days)

Task $j$	0	1	2	3	4	5	6	7	8
Project 1	0	3	5	6	2	3	3	4	0
Project 2	0	2	5	1	3	3	4	2	0

bounds of the parameters are  $J = 8$ ,  $R = 2$ ,  $D = 25$ , and  $C = 2$ .

Table 2 shows the mediating parameter  $q(d)$  used to simplify the coding in the relationship between two continuous and discrete variables and capture the numerical value of the period index as a parameter of the model.

Table 3 shows the amount of resource  $r$  used in activity  $j$  of project  $c$  within the running time of the project. The notation  $c/j$  denotes the project number/the activity number. For example, 1/2 represents activity 2 in project 1. The value of each cell shows the required amount of related resources for the activity. For example, activity 2 in project 1 needs two units of resource 1, four units of resource 2, and two units of resource 3.

Figures 1 and 2 illustrate the predecessor relationships between the activities of each project ( $relation(i, j, c)$ ) in the form of a node network. For example, the number 2 surrounded by a circle in Fig. 1 indicates that, in project 1, activity 2 is predecessor of activity 3.

Table 4 shows the rent of each resource paid to the resource owner in the second column and the total cost paid by the contractor for receiving the resource  $r$  from the other contractor in the third column, respectively. In the cooperative setting of the model, the fourth column shows the revenue that the employer pays to the resource-owning contractor per each run of the game at each time period. Columns 5 and 6 represent the amounts of limited resources contained in projects 1 and 2, respectively.

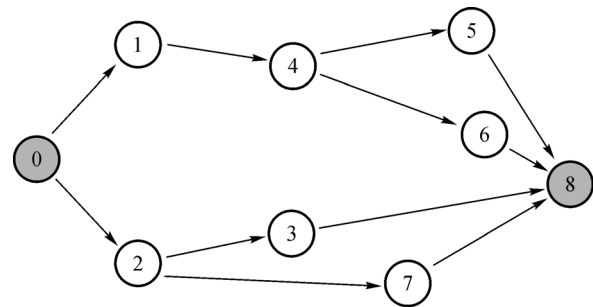
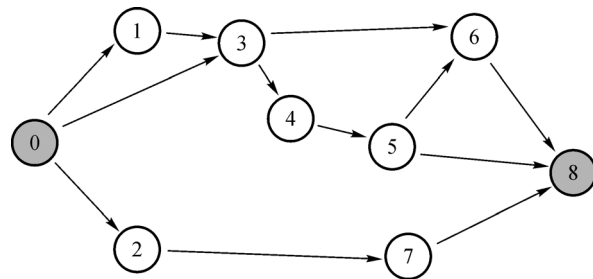
Given that  $h(r)$  includes the side costs of the resource exchange, its value is larger than  $s(r)$ .  $k(r)$  is considered for the revenue owing to the promotion of the cooperative implication of the projects. In addition, the employer can determine the optimal revenue for particular periods and resources by using indices  $r$  and  $d$ . Table 5 presents parameter  $n(c, c')$  and the respective amount of money that the employer pays to contractor  $c'$  for the early completion of project  $c$  every day compared with the details of the initial optimum date.

## 5.2 Scalars

The two parameters of  $gg_1$  and  $gg_2$  in the studied case are determined as 17 and 18, respectively. These scalars are the corresponding optimum times of completion for the first

**Table 3** Amount of resource  $r$  used in activity  $j$  of project  $c$ 

$c/j$	$r_1$	$r_2$	$r_3$	$c/j$	$r_1$	$r_2$	$r_3$
1/0	0	0	0	2/0	0	0	0
1/1	3	2	1	2/1	1	1	2
1/2	2	4	2	2/2	4	3	0
1/3	3	1	2	2/3	1	4	1
1/4	4	3	1	2/4	2	0	3
1/5	2	0	3	2/5	2	2	3
1/6	1	1	1	2/6	1	2	1
1/7	3	1	1	2/7	3	4	0
1/8	0	0	0	2/8	0	0	0

**Fig. 1** Node network activities and their interrelationships in project 1.**Fig. 2** Node network activities and their interrelationships in project 2.

stages of projects 1 and 2, which had been individually scheduled using the resources of a contractor.

**Table 2** Mediating parameter  $q(d)$ 

$q(d)$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$d$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

**Table 4** Parameters  $s(r)$  &  $h(r)$  &  $k(r)$  &  $b(r, c)$ 

Resource $r$	$s(r)$	$h(r)$	$k(r)$	$b(r, 1)$	$b(r, 2)$
$r_1$	100	150	30	6	4
$r_2$	300	400	50	4	5
$r_3$	400	550	60	5	4

**Table 5** Parameter  $n(c, c')$ 

	Contractor 1	Contractor 2
Project 1	10500	5500
Project 2	4500	9000

### 5.3 Results obtained using the initial model

The purpose of this section is to validate the mathematical performance of the proposed model based on the investigation of the achieved results. In other words, our aim is to show that the results are logical and applicable for real cases. The Gantt chart of the first project shown in Table 6 represents the results obtained for variable  $x(j, d)$ . The number 1 surrounded by a black circle corresponds to the schedule of activity 4 at time period 13. The blue rectangle corresponds to the implementation of activity 7, which starts from time period 9 and continues until time period 12, with its implementation expected to be completed at the end of time period 12. Table 7 shows the same information about project 2 in the initial model. The arrows indicate the predecessor relationships among the activities.

#### 5.4 Comparison of the durations of the projects by using the proposed model via the classical models

The aim of this section is to compare the required durations of the projects once we employ the proposed model against the classical scheduling models. Our aim is to show that the duration of the projects will decrease once we apply the proposed model. Table 8 presents the results obtained for the  $x(j, d)$  variable from projects 1 and 2, respectively, in

the proposed model, along with the Gantt chart of the first project. The number 1 surrounded by a black circle corresponds the schedule of activity 4 of project 1 at time period 4. The blue rectangle represents the implementation of activity 7, which starts from time period 11 and continues until time period 14, and its implementation is expected to be completed at the end of time period 14.

In Fig. 3, we compare our proposed model against the previous one in terms of project duration. Four indices are utilized for the required time to complete projects 1 and 2, respectively, the total required time to complete all of the projects, and the required time to complete the plan. The comparative results between the proposed model and the conventional project scheduling model suggest the following:

- 1) The time needed to complete project 1 has decreased from 17 to 14 days, that is, by 3 days or 17.6%.
- 2) The time needed to complete project 2 has decreased from 18 to 13 days, that is, by 5 days or 27.8%.
- 3) Overall, in the two projects, the total of 35 working days in the initial model has reduced to a total of 27 working days in the developed model, that is, by 8 days or 22.9%.
- 4) The time needed to complete the plan (i.e., when the plan is completed, both projects are also completed) has decreased from 18 working days in the initial model to 14 working days in the developed model, that is, by 4 days or 22.2%.

The above reductions in project duration are useful for companies in minimizing the cost of hiring day-workers, maximizing the possibility to participate in more tender offers, and improving job efficiency in the company.

### 5.5 Comparison of resource stagnation times of both projects between the initial and developed models

Our aim in this section is to compare the required resources for the projects when we employ the proposed model against the classical scheduling models. We will show that the required resources for the two projects will decrease when the proposed model is applied to reduce the stagnation time of the resources and increase the efficiency

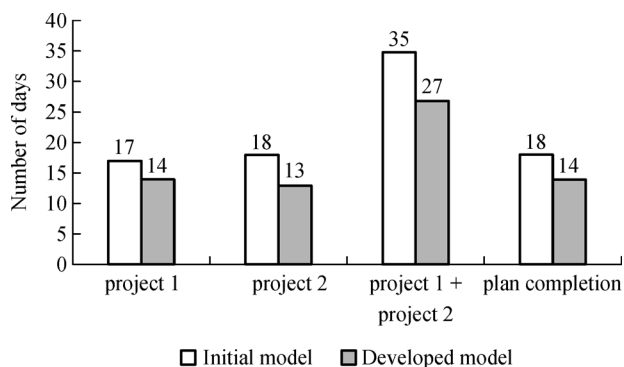
**Table 6** Results obtained for the  $x(j, d)$  variable from project 1 in the initial model

**Table 7** Results obtained for the  $x(j, d)$  variable from project 2 in the initial model

$j$	$d$																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0																				
1						1	1													
2	1	1	1	1	1															
3								1												
4									1	1	1									
5												1	1	1						
6															1	1	1	1		
7																				
8																				

**Table 8** Results obtained for the  $x(j, d)$  variable from projects 1 and 2, respectively, in the developed model

$clj$	$d$																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1/0																				
1/1	1	1	1																	
1/2				1	1	1	1	1												
1/3									1	1	1	1	1	1						
1/4				1	1															
1/5						1	1	1												
1/6									1	1	1									
1/7											1	1	1	1						
1/8																				
2/0																				
2/1	1	1																		
2/2						1	1	1	1	1										
2/3			1																	
2/4				1	1	1														
2/5							1	1	1											
2/6									1	1		1	1							
2/7													1	1						
2/8																				

**Fig. 3** Comparison of time completion of the two projects in the development of the initial model.

of the resources. Tables 9 and 10 show the surplus (stagnation time) of resource  $r$  at each time period  $d$  in projects 1 and 2, respectively, in the initial model. Table 11 shows the total surplus of resource  $r$  at each time period  $d$  in both projects in the developed model.

Figure 4 compares the bedtimes of the resources available in both projects between the initial and developed models. The bedtime of resource type 1 has decreased from 63 days in the initial model to 29 days in the developed model. In both models, the amounts of used resources are equal; however, the completion times of both projects in the developed model are shorter than those in the classical model. Thus, the total resource-days available in both projects in the initial model differ from those in the developed model. The results are computed by (working days  $\times$  whole project  $\times$  resources available in the project).

**Table 9** Surplus of resource  $r$  at each time period  $d$  in project 1 in the initial model

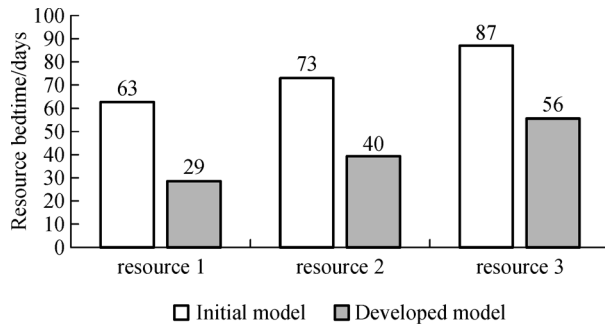
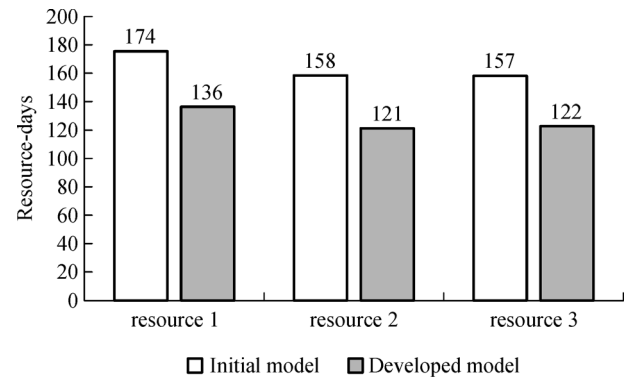
Resource	$d$																				Sum
$r$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
$r_1$	4	4	4	4	4	3	0	0	0	0	0	0	2	2	3	3	3				36
$r_2$	0	0	0	0	0	2	1	1	2	2	2	2	1	1	3	3	3				23
$r_3$	3	3	3	3	3	4	2	2	2	2	2	2	4	4	1	1	1				42

**Table 10** Surplus of resource  $r$  at each time period  $d$  in project 2 in the initial model

Resource	$d$																				Sum
$r$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
$r_1$	0	0	0	0	0	0	0	3	2	2	2	2	2	2	3	3	3	3			27
$r_2$	2	2	2	2	2	0	0	1	5	5	5	3	3	3	3	3	3	3			47
$r_3$	4	4	4	4	4	2	2	3	1	1	1	1	1	1	3	3	3	3			45

**Table 11** Total surplus of resource  $r$  at each time period  $d$  in projects 1 and 2 in the developed model

Resource	$d$																				Sum
$r$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
$r_1$	6	6	6	2	2	0	0	0	0	1	2	0	0	4							29
$r_2$	6	6	3	2	2	2	0	0	4	2	4	1	1	7							40
$r_3$	6	6	7	3	3	1	1	1	3	5	4	5	5	6							56

**Fig. 4** Comparison of resource bedtimes available in both projects between the initial and developed models.**Fig. 5** Comparison of the total resource-days available in both projects between the initial and developed models.

As shown in Fig. 5, the number of resource-days available in both projects for resource type 1 has decreased from 174 in the initial model to 136 in the developed model, indicating a 38 resource-days reduction (21.8%), which for resource type 2 has decreased by 37 resource-days (23.4%), while that for resource type 3 has decreased by 35 resource-days (22.3%). These numbers are further converted into revenues for contractors in terms of their early completion of projects, and the contractors can rent their freed-up resources or employ them in other projects.

The reductions in bedtime and the increasing availability

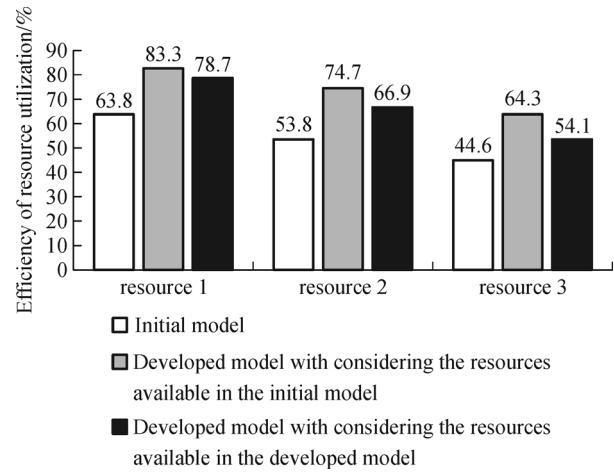
of resources can considerably benefit companies in terms of minimizing operational costs for resources, maximizing the life span of resources, maximizing the power of company to participate in more tender offers, and increasing the competitive advantage of the company.

The efficiency of resource utilization is determined by dividing the number of the used resource-days in the whole project by the total number of resource-days. We perform a fair comparison between the initial and developed models

by considering both of their number of available resources. As shown in Fig. 6, the efficiency of the resource utilization in both projects for resource type 1 is 63.8% in the initial model and 83.3% in the developed model (i.e., 19.5% increase). The increases are 20.9% for resource type 2 and 19.7% for resource type 3, respectively. Thus, the value of the increased efficiency in renewable resource utilization is highly significant. In addition, such benefits will enable companies to reduce their costs associated with their operational cash flow costs.

### 5.6 Nominal and actual revenue of contractors in the developed model

Our aim in this section is to compute the actual revenues of the contractors by using the developed model, which represents the actual benefits, including certain new revenues besides nominal revenue. The new benefits are not considered in the previous research, and they are created according to the released resources because of the projects' much shorter duration. These advantages may persuade contractors to participate and cooperate. As



**Fig. 6** Comparison of efficiencies in resource utilization in both projects between the initial and developed models.

mentioned previously, the revenues calculated by the contractors in the model are nominal revenues, which can be obtained from the following formulas:

$$in1_{\text{marginal}} = \left\{ \begin{aligned} & n(1, 1) \times (gg_1 - g(1)) + n(1, 2) \times (gg_2 - g(2)) + \sum_{r=1}^R \sum_{d=0}^D s(r) \times w(d, r, 2) \\ & - \sum_{r=1}^R \sum_{d=0}^D h(r) \times w(d, r, 1) + \sum_{r=1}^R \sum_{d=0}^D k(r) \times v(d, r, 2) \end{aligned} \right\} = 19199,$$

$$in2_{\text{marginal}} = \left\{ \begin{aligned} & n(2, 2) \times (gg_2 - g(2)) + n(2, 1) \times (gg_1 - g(1)) + \sum_{r=1}^R \sum_{d=0}^D s(r) \times w(d, r, 1) \\ & - \sum_{r=1}^R \sum_{d=0}^D h(r) \times w(d, r, 2) + \sum_{r=1}^R \sum_{d=0}^D k(r) \times v(d, r, 1) \end{aligned} \right\} = 28879.$$

However, the actual revenues of the contractors may be much larger than these values, as the revenue associated with the freed-up resources is usually not significant for the contractor.

$$\begin{aligned} in1_{\text{real}} &= in1_{\text{marginal}} + s(r) \times b(r, 1) \times (gg_1 - g(1)) \\ &= 19199 + (100 \times 6 \times 3) + (300 \times 4 \times 3) \\ &\quad + (400 \times 5 \times 3) = 30599, \end{aligned}$$

$$\begin{aligned} in2_{\text{real}} &= in2_{\text{marginal}} + s(r) \times b(r, 2) \times (gg_2 - g(2)) \\ &= 28879 + (100 \times 4 \times 5) + (300 \times 5 \times 5) \\ &\quad + (400 \times 4 \times 5) = 46379. \end{aligned}$$

### 5.7 Game performance in the model for each contractor

Our aim in this section is to evaluate the performance of

the developed games and the exchanges of resources. Tables 12 and 13 present the performed game and the resource exchange performance. The values obtained for the variable  $v(d, r, c)$  for players 1 and 2 are both determined. A value of 1 for variable  $v(d, r, 1)$  in Table 12 indicates that player 1 at time period  $d$  has used more resource  $r$  than his resource  $r$ , and player 2 in the game has shared a number of his resource  $r$  with player 1. The results indicate that the game is possible for player 2 in three periods: Player 2 should lend his resource type 2 to player 1 at periods 4 and 5, and lend his resource type 1 to player 1 at period 11. A value of 1 for variable  $v(d, r, 2)$  in Table 13 indicates that player 2 at period  $d$  has used more resource  $r$  than his resource  $r$ , and player 1 in the game has shared a number of his resource  $r$  with player 2. The results indicate that the game is possible for player 1 in seven periods: Player 1 should lend his resource type 1 to player 2 at periods 6–10, and share his resource type 2 to player 1 at periods 12 and 13.

**Table 12** Performed game or resource exchange taken by the second contractor  $v(d, r, 1)$ 

Resource	$d$																			
$r$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$r_1$	0	0	0	0	0	0	0	0	0	0	1	0	0	0						
$r_2$	0	0	0	1	1	0	0	0	0	0	0	0	0	0						
$r_3$	0	0	0	0	0	0	0	0	0	0	0	0	0	0						

**Table 13** Performed game or resource exchange taken by the first contractor  $v(d, r, 2)$ 

Resource	$d$																			
$r$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$r_1$	0	0	0	0	0	1	1	1	1	1	0	0	0	0						
$r_2$	0	0	0	0	0	0	0	0	0	0	0	1	1	0						
$r_3$	0	0	0	0	0	0	0	0	0	0	0	0	0	0						

### 5.8 Amount of resources exchanged in the optimum strategy of the contractors

Our aim in this section is to illustrate the amount of exchanged resources at each project period according to the proposed model. The resources should be exchanged between the contractors during the determined periods. In this manner, we can reach the benefits of the model, as described in the previous sections. Variable  $w(d, r, c)$  represents the number of resources exchanged at each game. Table 14 shows the number of resource  $r$  that player 2 has lent to player 1 at period  $d$ . The results denote that player 1 has borrowed three sets of resource type 2 from player 2 at periods 4 and 5. Moreover, player 1 has borrowed (rented) one additional resource type 1 from player 2 at period 11.

Table 15 illustrates the number of resource  $r$  that player 1 has lent to player 2 at time period  $d$ . The results demonstrate that player 2 has borrowed two resource type 1 from player 1 at periods 6–9, then borrowed (rented) one resource type 1 from player 1 at period 10 and one resource type 2 from player 1 at periods 12 and 13.

According to Tables 12–15 and Fig. 7, a successive dynamic game runs in the model, which consists of four games as follows:

Game 1: At the end of period 3 and the beginning of period 4, player 2 should decide whether or not to give three sets of his resource type 2 for periods 4 and 5 to player 1.

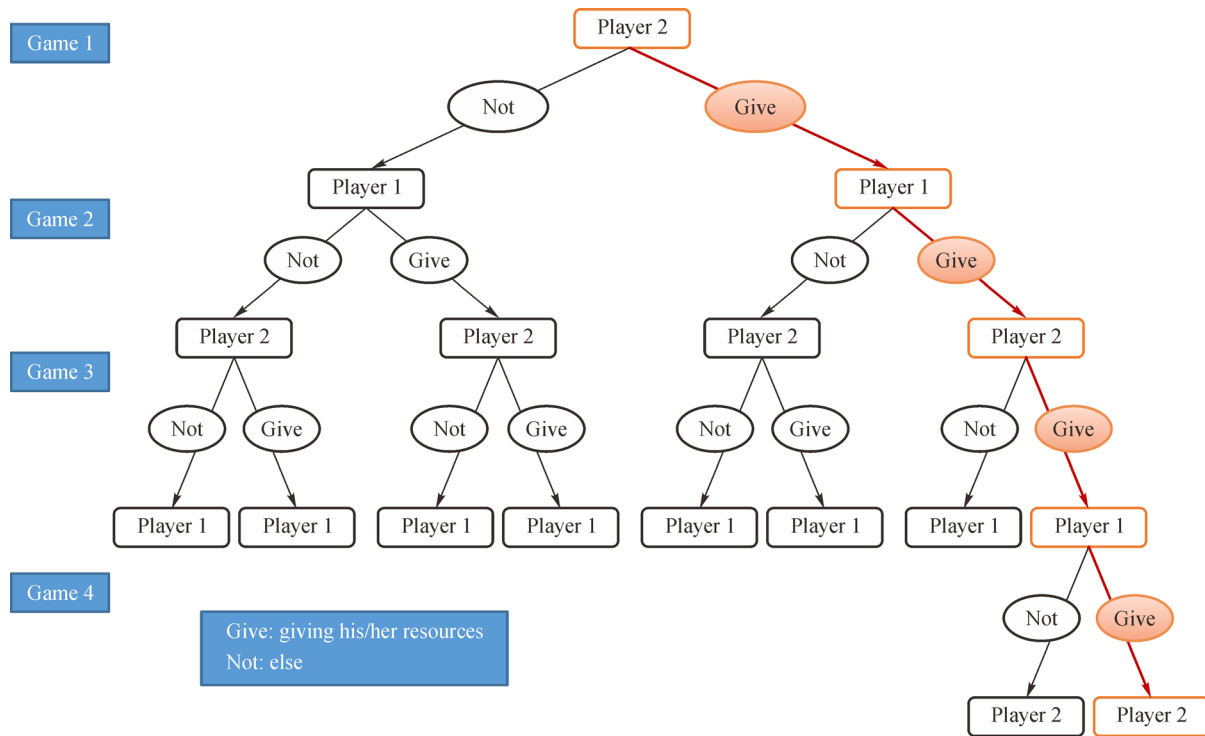
Game 2: At the end of period 5 and the beginning of period 6, player 1 should decide whether or not to give two and one sets of his resource type 1 for periods 6–9 and 10,

**Table 14** Number of resource  $r$  that player 2 has lent to player 1 at time period  $d$ ,  $w(d, r, 1)$ 

Resource	$d$																			
$r$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$r_1$	0	0	0	0	0	0	0	0	0	0	1	0	0	0						
$r_2$	0	0	0	3	3	0	0	0	0	0	0	0	0	0						
$r_3$	0	0	0	0	0	0	0	0	0	0	0	0	0	0						

**Table 15** Number of resource  $r$  that player 1 has lent to player 2 at time period  $d$ ,  $w(d, r, 2)$ 

Resource	$d$																			
$r$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$r_1$	0	0	0	0	0	2	2	2	2	1	0	0	0	0						
$r_2$	0	0	0	0	0	0	0	0	0	0	0	1	1	0						
$r_3$	0	0	0	0	0	0	0	0	0	0	0	0	0	0						



**Fig. 7** Schematic view of the step-by-step game between two contractors and the strategy of each contractor. The optimum strategy of the model is shown in red.

respectively, to player 2.

Game 3: At the end of period 10 and the beginning of period 11, player 2 should decide whether or not to give one set of his resource type 1 for period 11 to player 1.

Game 4: At the end of period 11 and the beginning of period 12, player 1 should decide whether or not to give one set of his resource type 2 for periods of 12 and 13 to player 2.

### 5.9 Discussion

We propose in this study an approach to schedule grand projects based on the resource sharing mechanism by using game theory. In real-world problems, the procurement managers of the contractors need to be highly consistent when planning a resource sharing program. Constant time is assumed in sending and sharing resources between contractors. Thus, managing the time of resource sharing is regarded significant, and the procurement managers play a key role on this matter. In addition, the financial managers of the contractors should be consistent in the cost of depreciation of the machineries. In other words, the managers should have a common understanding of the operational costs of the shared resources during the cooperation. However, the education process of laborers in terms of using shared resources and practicing communication skills to transfer professional knowledge should be considered.

## 6 Conclusions

We propose in this study a scheduling framework for grand projects, in which several contractors work concurrently on their sub-projects. Game theory is used to improve the project's duration, and the resource sharing of machineries are considered, while the minimum increase in cost is allowed. If a player in the designed game has selected a non-optimum strategy, then the plan for the succeeding time periods can be immediately updated. The employed game theory can analyze the scheduling of resources among multiple contractors. Given the NP-hardness of the mathematical model, the Benders decomposition method is applied to solve the problem in the large-scale examples. The results of the model indicate a considerable reduction of project's duration against those of conventional models. A comparison between the developed model and the conventional model in terms of project scheduling indicates that new conditions can decrease cost and completion time, and employers and contractors are more motivated to cooperate and engage in cooperative management. The limitations of our research are as follows: 1) lack of adherence to the employers, 2) difficulty and cost of controlling and monitoring the game, and 3) cannot be easily applied to projects that include "prioritize activities" and resource limitations.

As for the future direction of this research, we recommend the following points: 1) use of multi-objective



programming to minimize the completion time of projects and maximize the revenues, 2) optimization of assigned revenues to contractors from the employer, and 3) consideration of uncertainty scenarios in selecting the strategy of the contractors.

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