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RESEARCH ARTICLE

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Two-level uncapacitated lot-sizing problem considering the financing cost of working capital requirement

Abstract During financial crisis, companies constantly need free cash flows to efficiently react to any uncertainty, thus ensuring solvency. Working capital requirement (WCR) has been recognized as a key factor for releasing tied up cash in companies. However, in literatures related to lot-sizing problem, WCR has only been studied in the single-level supply chain context. In this paper, we initially adopt WCR model for a multi-level case. A two-level (supplier–customer) model is established on the basis of the classic multi-level lot-sizing model integrated with WCR financing cost. To tackle this problem, we propose sequential and centralized approaches to solve the two-level case with a serial chain structure. The ZIO (Zero Inventory Ordering) property is further confirmed valid in both cases. This property allows us to establish a dynamic programming-based algorithm, which solves the problem in $O(T^4)$. Finally, numerical tests show differences in optimal plans obtained by both approaches and the influence of varying delays in payment on the WCR of both actors.

Keywords two-level ULS problem, lot-sizing, working capital requirement, ZIO property, infinite production capacity

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1 Introduction

Tactical production planning is one of the major decision processes in production management as emphasized in the MRP II (Manufacturing Resource Planning) methodology. Tactical planning aims to fulfill customers' demands by determining the quantities to be manufactured while minimizing logistic costs, such as holding, setup, and production costs. To tackle this problem, several mathematical models have been designed; among them, "lot-sizing" models have been recognized for their efficiency (Drexl and Kimms, 1997). In recent years, literatures of supply chain management, which followed the original work of Babich and Sobel (2004), have become aware that financial and operational problems are imbricated. In addition, simultaneously optimizing the two dimensions can improve companies' global performance in a more efficient way (Peng and Zhou, 2019).

Extending the tactical plan process by considering financial aspects and optimizing logistic costs and their financial impacts seems promising. This process can reduce bankruptcy risk by finding the trade-off between the logistic and financial costs generated by operation decisions made at the tactical supply chain management. Therefore, this study proposes to deal with tactical planning at a multi-level supply chain with a serial structure while considering different financial aspects, such as financing of Working Capital Requirement (WCR) and the net discounted value.

Previous works focused on a single-level model either with a constant demand (Harris, 1913) or a variable demand (Wagner and Whitin, 1958). In this study, we consider the multi-level lot-sizing problem (MLLP), which has been elaborated to plan end items and components, owing to their bill of material under infinite capacities assumption based on MRP (Material requirements planning) philosophy. This mathematical formulation can model the considered supply chain. Moreover, this research aims to extend the MLLP model by integrating financial costs and the profit of each site. We focus on profit maximization problem, given that the profit is the difference between the revenue and the sum of the logistic and financial costs.

To address this complex problem, we initially study the two-level problem by extending the work of Bian et al. (2018), including financial costs, with concept of WCR and Net Present Value (NPV), in the case of a serial structure (see Fig. 1). This problem is called $2ULS_{P(WCR)}$ (uncapacitated lot-sizing problem), wherein we consider a supply chain that is composed of two sites and that extends the previous single-level model. In this system, a supplier purchases and processes raw materials to produce intermediate goods in site S_0. These goods are delivered to a manufacturer for further processing in site S_1 to satisfy the external demand. Only one type of item is manufactured in each site, which is denoted as P_0 in S_0 and P_1 in S_1. Additional assumptions are presented in the paragraphs below.

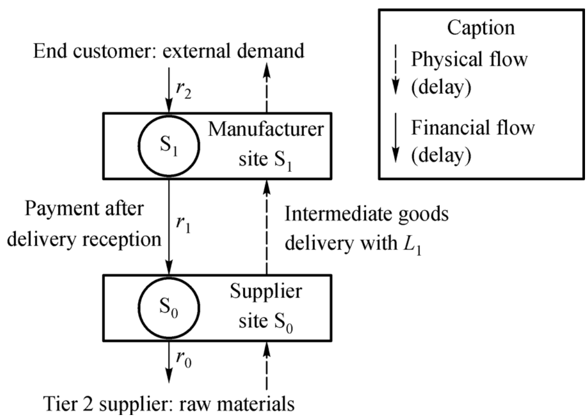

Fig. 1 Physical and financial flows in a two-level supply chain.

Each site is characterized by the following:

• Logistic costs. Such costs include purchasing (denoted by a_i, $i\in\{0, 1\}$), setup (denoted by s_i, $i\in\{0, 1\}$), production (denoted by p_i, $i\in\{0, 1\}$), and inventory holding costs (denoted by h_i, $i\in\{0, 1\}$). Generally, the purchasing unit cost of the manufacturer should be higher than the total unit cost for its production of the supplier (i.e., including all four types of costs).

• Term of payment. One of the differences between single-level case and two-level case is the payment delay between site $i-1$ and site i (denoted as r_i). In the two-level case, the supplier receives the payment from the manufacturer after $L_1 + r_1$ periods, which include the delivery delay L_1 and payment delay r_1 between these sites. Furthermore, no particular index is given for the final customer, who commands the external demand, and the tier 2 supplier, who provides the raw material. Payment delay from the final customer to the manufacturer is denoted by r_2, whereas the payment from the supplier to the tier 2 supplier is denoted by r_0. Moreover, the delivery delays, the L of raw material and the finished product are not considered in this problem, that is, $L_0 = L_2 = 0$ (they remain in the mathematical formulation for not losing generality).

• Financial aspects. The same financial aspects (discount and interest rates) in the previous single-level model are considered in this case. The discount rate is denoted by α_i, $i\in\{0, 1\}$, whereas the interest rate (the rate of financing the WCR) is denoted by β_i, $i\in\{0, 1\}$.

Hofmann et al. (2011) indicated that the buying company often has a lower capital cost rate than its supplier, resulting from a shortened cash-to-cash cycle. In practice, the unit sales price of supplier (i.e., the unit purchasing cost of manufacturer) is usually greater than the average cost per product, including all logistic and financial costs, to gain profit. Furthermore, the supplier frequently has a lower inventory carrying rate since its goods are warehoused in large quantities and scaling effects are achieved (Hofmann et al., 2011). Thus, the inventory holding cost is greater in S_1 than in S_0 (i.e., $h_0 < h_1$). We assume that $r_1 \geqslant r_2$ for this problem, which implies that the manufacturer takes the dominant role in the chain. We also assume that the interest rates are equal in the centralized approach. In conclusion, the two-level model is established under the following assumptions:

Production aspects

• No replenishment and production delays are anticipated, given that they are negligible compared with the period duration;

• Demand should be met on time (no backlogging);

• The initial and final stocks of all items are defined as zero;

• One unit of item is manufactured using one unit of component or raw material;

• Delivery delays of raw materials and finished products are not considered;

• Intermediate goods are only held in the inventory of the supplier;

• Inventory holding unit cost is greater in the manufacturer level than the one in the supplier level.

Financial aspects

• The payment of the manufacturer to the supplier for the intermediate goods is executed after the receipt of the goods;

• Payments of all logistics and financial costs are made at initial periods;

• The margin caused by the sale of selling products is not used for financing WCR;

• Products in the same lot uniformly share the setup cost. However, the inventory holding cost is measured for each product on the basis of its total holding time in the inventory;

• The purchasing unit cost of one level should be higher than the unit cost for producing one unit at the lower (adjacent) level;

• Discount rate is greater at the supplier level than that at the manufacturer level;

• Payment delay from the manufacturer is shorter or equal to the one from the supplier;

• The interest rates of the two levels are assumed equal in the centralized approach.

2 Literature review

Working capital management (WCM) has been recognized for its promising potential to improve supply chain performance (Timme and Williams-Timme, 2000). How WCM affects company performance and profitability has been revealed at the national and sector levels (Lind et al., 2012; Enqvist et al., 2014). In our work, we study the impact of cash flows on the production plans within an enterprise network. In this section, the classic linear formulation of MLLP and the resolution approaches in the lot-sizing literature are presented. Moreover, the definition of WCR in the fields of finance and accounting is summarized.

2.1 MLLP

MLLP can be modeled as a mixed integer linear program (MILP). Table 1 describes the notations of the parameters and the decision variables. The gozinto coefficient a_{ij} is equal to zero if item i is not an immediate successor of item j. Otherwise, the quantity of item j should be equal to that required for producing one item i.

Table 1 Notation for MLLP

Parameters	
T	Number of periods
N	Number of items
d_{it}	Customer's demand for item i at period t
a_{ij}	Gozinto coefficient
h_i	Inventory holding cost for item i
s_i	Setup cost for item i
I_{i0}	Initial inventory for item i
M	Big number
Decision variables	
X_{it}	Production quantity for item i at period t
I_{it}	Inventory for item i at the end of period t
Y_{it}	Binary variable that indicates whether a setup for item i occurs at period t

The objective function is the sum of setup and inventory holding costs as formulated in the following expression:

$$\text{Max} \sum_{t=1}^{T} \sum_{i=1}^{N} (h_i \cdot I_{it} + s_i \cdot Y_{it}), \tag{1}$$

$s.\ t.$

$$X_{it} \leqslant M \cdot Y_{it},\ \forall (i,\ t) \in [1,\ N] \times [1,\ T], \tag{2}$$

$$I_{it} = I_{i(t-1)} + X_{it} - d_{it} - \sum_{j=1}^{N} a_{ij} \cdot X_{it},$$

$$\forall (i,\ t) \in [1,\ N] \times [1,\ T], \tag{3}$$

$$(I_{it},\ X_{it}) \in \mathbb{N}^2,\ Y_{it} \in \{0, 1\},\ \forall (i,\ t) \in [1,\ N] \times [1,\ T]. \tag{4}$$

Equation (2) determines whether a setup for the production of item i occurs at period t. Equation (3) represents inventory balance. Equation (4) represents the integrity constraints.

2.2 Resolution methods for MLLP in the literature

Sequential approach of the MLLP problem is one of the first approaches proposed on the basis of the MRP method, which plans the production level by level. Early works on the sequential application of single-level algorithm (including Wagner-Whitin algorithm for ULS) have been investigated by Yelle (1979) and Veral and LaForge (1985). Later, approximate applications of the MLLP problem are proposed by integrating the logistics cost of the end item into its components. In certain cases, cost modifications are proposed (Black-burn and Millen, 1982; Bookbinder and Koch, 1990; Dellaert and Jeunet, 2003).

In the non-distributed MLLP literature, the centralized approach remains the main stream since the early works of Zangwill (1968; 1969). The author has optimally solved the problem with serial BOM (bill of material) by proposing dynamic programming-based algorithms. Other optimal algorithms include the assembly-structure-based method of Crowston and Wagner (1973), and branch and bound algorithms of Afentakis et al. (1984) and Afentakis and Gavish (1986). However, the optimal solution can be obtained for small instances, given that the complexity of the MLLP problem is NP (non-deterministic polynomial)-hard (Steinberg and Napier, 1980). The large-sized MLLP problem becomes further difficult to investigate for satisfactory results with reasonable computational effort. For this reason, metaheuristic algorithms have been largely developed for the efficiency of solving this problem in complex realistic structure cases. Dellaert and Jeunet (2000) established a hybrid genetic algorithm for this problem with a general product structure. Tang (2004) proposed the simulated annealing algorithm for assembly structure and one finished item. Han et al. (2009) and Deroussi and Lemoine (2009) also suggested particle swarm optimization algorithms for MLLP with an assembly structure. A maximum–minimum ant colony optimization algorithm is proposed for serial, assembly, and general structure problems in the study of Pitakaso et al. (2007). Moreover, a neighborhood search based meta-heuristic technique has been developed by Xiao et al. (2011a) for small- and medium-sized problems (Xiao et al., 2011b; 2012; 2014). Certain studies focus on collaborations among facilities, such as Fink (2004) and Homberger and Gehring (2010).

To fill the research gap, our work is the first to integrate the WCR financing cost into the MLLP problem. The two-level serial chain case is initially considered in this study, and we proved that this new problem can be solved in polynomial time with assumptions.

3 MLLP with WCR financing cost

3.1 WCR model in the multi-level context

Bian et al. (2018) defined WCR as “the minimum financial resources needed for firms in order to run their business activities”. More specifically, it is the minimum amount of financial resources to cover operational costs before receiving client payments for goods and/or services. The WCR is mainly caused by a mismatch between cash inflows (accounts receivable) and cash outflows (accounts payable) associated with the total production and commercial cycle as presented in Guez (2014). WCR is measured as the sum of accounts payable, accounts receivable, and inventories (Hofmann and Kotzab, 2010). WCR has two components, namely, operating WCR (OWCR) and non-operating WCR (NOWCR). NOWCR is generated by the time mismatch of cash flows, which are related to non-operating activities (debt on investment, dividends to be collected or to be disbursed, exceptional events, among others). Given its marginal importance, NOWCR has no particular economic significance. In contrast to the recurrent OWCR, NOWCR is difficult to predict and analyze and is occasionally event-based. Therefore, we only consider OWCR and regard it as WCR in this work. The operating cycle corresponds to the regular and recurring company activities. This regularity entails permanent financial consequences given that they are commonly renewed. Considering this dynamic in the financial analysis is ensured by the notion of financing need for operation or WCR. Thus, WCR is considered a financial need caused by company activities, which require financial resources to cover.

WCR is justified by the following simple principles:

• A receivable (or payable), although acquired and certain, is not usually paid immediately by the customer (to the supplier);

• A stock is not sold immediately, and the products remain in stock for a period before sold.

These different gaps generate a financial need for companies that should be financed either by the settlement period negotiated with the supplier, by the working capital, or by the treasury. Consequently, WCR can be expressed by the following formula:

$$\begin{aligned}\text{WCR} = {} & \text{Account receivable} + \text{Inventory value} \\ & - \text{Account payable.}\end{aligned}$$

Figure 2 illustrates WCR through a small example (Pu. stands for Purchasing, Pr. stands for Production, I.H. stands for Inventory Holding, and De. is for Delivery). The payment delays between the manufacturer and its supplier and between the end-customer and the manufacturer are considered in one period. A production decision taken at period 1 implies the immediate payment of production costs. Purchasing costs are paid at period 2 due to the payment delay. Thus, a product is sold at period 2. Inventory holding costs are paid at periods 1 and 2. The manufacturer receives the money at period 4 due to payment delay. Thus, the manufacturer must finance the logistics costs from periods 1 to 4.

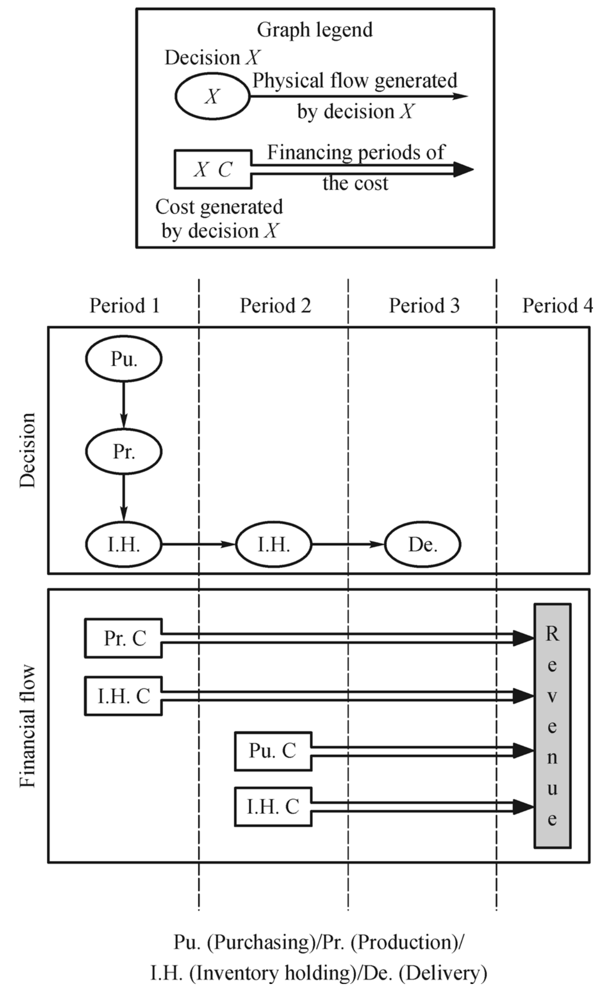

Fig. 2 Illustration of WCR in the context of a lot-sizing problem (Bian et al., 2018).

Table 2 presents that the notations of parameters and decision variables adopted in the two-level case are similar to that in the single-level model. In such a case, parameter i

Table 2 Notations for the $2ULS_{P(WCR)}$

Parameters	
T	Number of periods
d_{it}	Customer's demand for site i at period t
v_i	Unit product price for item i
a_i	Unit raw material cost for item i
h_i	Inventory holding cost for item i
s_i	Fixed setup cost for item i
p_i	Unit production cost for item i
r_i	Delay in payment from site i to site $i-1$
L_i	Delivery delay from site $i-1$ to site i
α_i	Discount rate per period of site i
β_i	Interest rate for financing WCR of site i
Decision variables	
Q_{it}	Total production quantity at site i in period t
X_{itk}	Production quantity in period t for satisfying (a part of) demand in period k of site i
I_{it}	Inventory for item i at the end of period t
Y_{it}	Binary variable, which indicates whether a setup for item i occurs at period t

can take three values, namely, 0 for the supplier, 1 for the manufacturer, and 2 for the external customer. Bian et al. (2018) explained the reason of using the disaggregation variable.

3.2 Mathematical formulation of WCR

During each period, WCR is composed of the same terms as the single-level case for covering the purchasing (purch), setup (setup), production (prod), and inventory holding (inv) costs. The formulation of WCR in this case is similar to the previous case to cover the different logistics costs associated to the disaggregated production quantity, X_{itk} (where $i\in\{0, 1\}$, $t\in[0, T]$, and $k\in[t, T]$). For instance, WCR_{itk}^{purch} represents the WCR for financing the purchasing cost related to X_{itk}. Without loss of generality, we consider the beginning of the supplier's production as the beginning of the global planning horizon. For this reason, the delivery delay to site i can postpone its planning in the global planning horizon and add discount effects to the incoming and outgoing cash flows of site i. For instance, the first term in the formulation of the manufacturer's WCR, $\frac{1}{(1+\alpha_i)^{L_i}}$, reflects the discount effect of this backward shift of planning horizon for L_1 periods. The WCR of the supplier and of the manufacturer are expressed by the following equations, where $i\in\{0, 1\}$, $t\in[0, T]$, and $k\in[t, T]$. Accordingly, the supplier's WCRs are written as:

$$WCR_{0tk}^{\text{purch}}=a_0\cdot X_{0tk}\cdot\left(\sum_{j=t+r_0}^{T}\frac{1}{(1+\alpha_0)^j}-\sum_{j=k+L_1+r_1}^{T}\frac{1}{(1+\alpha_0)^j}\right),\tag{5}$$

$$WCR_{0tk}^{\text{prod}}=p_0\cdot X_{0tk}\cdot\sum_{j=t}^{k+L_1+r_1-1}\frac{1}{(1+\alpha_0)^j},\tag{6}$$

$$WCR_{0tk}^{\text{setup}}=\frac{s_0\cdot Y_{0t}}{Q_{0t}+1-Y_{0k}}\cdot X_{0tk}\cdot\sum_{j=t}^{k+L_1+r_1-1}\frac{1}{(1+\alpha_0)^j},\tag{7}$$

$$WCR_{0tk}^{\text{inv}}=h_0\cdot X_{0tk}\cdot\sum_{w=t}^{k-1}\sum_{j=w}^{k+L_1+r_1-1}\frac{1}{(1+\alpha_0)^j}.\tag{8}$$

The manufacturer's WCRs are formulated as follows:

$$WCR_{1tk}^{\text{purch}}=\frac{1}{(1+\alpha_1)^{L_1}}\cdot a_1\cdot X_{1tk}\cdot\left(\sum_{j=t+r_1}^{T}\frac{1}{(1+\alpha_1)^j}-\sum_{j=k+r_2}^{T}\frac{1}{(1+\alpha_1)^j}\right),\tag{9}$$

$$WCR_{1tk}^{\text{prod}}=\frac{1}{(1+\alpha_1)^{L_1}}\cdot p_1\cdot X_{1tk}\cdot\sum_{j=t}^{k+r_2-1}\frac{1}{(1+\alpha_1)^j},\tag{10}$$

$$WCR_{1tk}^{\text{setup}}=\frac{1}{(1+\alpha_1)^{L_1}}\cdot\frac{s_1\cdot Y_{1t}}{Q_{1t}+1-Y_{1k}}\cdot X_{1tk}\cdot\sum_{j=t}^{k+r_2-1}\frac{1}{(1+\alpha_1)^j},\tag{11}$$

$$WCR_{1tk}^{\text{inv}}=\frac{1}{(1+\alpha_1)^{L_1}}\cdot h_1\cdot X_{1tk}\cdot\sum_{w=t}^{k-1}\sum_{j=w}^{k+r_2-1}\frac{1}{(1+\alpha_1)^j}.\tag{12}$$

Combining all these terms, the total WCR for site $i\in\{0, 1\}$ is given by the equation below:

$$WCR_i=\sum_{t=1}^{T}\sum_{k=t}^{T}(WCR_{itk}^{\text{purch}}+WCR_{itk}^{\text{prod}}+WCR_{itk}^{\text{setup}}+WCR_{itk}^{\text{inv}}),\tag{13}$$

and the financing cost of WCR_i is thus $\beta_i\times WCR_i$.

3.3 Mathematical formulation of objective functions

The objective functions of these approaches mainly depend on the total logistics and financial costs and the profit of each site. They can be written in a similar form to the single-level problem. The formulations of logistics cost for each site are composed of four components over the entire horizon. These components are computed using the following equations. Given that lead time is not considered in this problem (the reason is explained in Section 4), the logistics costs of the two sites can be expressed as follows:

$$LC_i^{\text{purch}} = a_i \sum_{t=1}^{T} \frac{Q_{it}}{(1+\alpha_i)^{t+r_i}}, \tag{14}$$

$$LC_i^{\text{prod}} = p_i \sum_{t=1}^{T} \frac{Q_{it}}{(1+\alpha_i)^{t}}, \tag{15}$$

$$LC_i^{\text{setup}} = s_i \sum_{t=1}^{T} \frac{Y_{it}}{(1+\alpha_i)^{t}}, \tag{16}$$

$$LC_i^{\text{inv}} = h_i \sum_{t=1}^{T} \sum_{k=t}^{T} \sum_{q=t}^{k-1} \frac{X_{it}k}{(1+\alpha_i)^{q}}, \tag{17}$$

where $i = 1$ signifies the manufacturer, and $i = 0$ represents the supplier. In sequential problem, the manufacturer's objective is prioritized, thereby leading to a maximization problem at S_1 level:

$$\text{Max } R_1 - (LC_1 + \beta_1 \times WCR_1). \tag{18}$$

This sub-problem can be simply solved using the approach proposed by Bian et al. (2018). The result is the input (demand) of the sub-problem at the supplier's level for another maximization problem, which can be resolved through the same approach:

$$\text{Max } R_0 - (LC_0 + \beta_0 \times WCR_0). \tag{19}$$

Therefore, we only focus on the centralized problem, which maximizes the total profit of both sites:

$$\text{Max} \sum_{i=0}^{1} [R_i - (LC_i + \beta_i \times WCR_i)]. \tag{20}$$

For both approaches, the constraints are similar to that of the classic MLLP problem.

4 Resolution method for centralized objective

In a centralized problem, we maximize the global profit of the two levels. Zangwill (1968) proposed a dynamic programming-based algorithm to solve the MLLP problem with serial BOM structure and a computation time of O (NT^4), where N is the number of levels in the problem. Zangwill (1968) modeled the problem in the form of a network with nodes presented in Fig. 3. Such a node (i, j) represents period i in the site at level j. Passing the node, the vertical arcs represent the production quantity (e.g., Q_{ij} is the production quantity in period i at level j), and the horizontal arc, I_{ij}, denotes the quantity of products held in inventory in period i at level j. Therefore, the production decision at the node (i, j) is considered in determining the value of $Q_{i(j+1)}$, which aims to satisfy the demand of level $j+1$. This decision is based on the upstream incoming flows, that is, Q_{ij} and $I_{(i-1)j}$. Moreover, this decision can determine the value of the other downstream flow, that is, I_{ij}. With the values of these arcs, we can calculate the setup and production costs in the function of Q_{ij} ($\forall i, j$) and the inventory holding cost in the function of I_{ij} ($\forall i, j$).

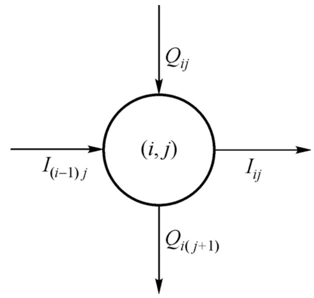

Fig. 3 A node and associated arcs in Zangwill's network presentation of the multi-level production problem.

The provenances of upstream flows cannot be precisely traced in this network representation. Therefore, we cannot obtain the information on the production time of the products in $I_{(i-1)j}$. Consequently, the associated WCR cannot be correctly measured. Thus, Zangwill's algorithm cannot be directly applied to this problem, and a new concept is required to address this problem. Therefore, we initially reveal that the ZIO property remains valid for this two-level problem. Subsequently, the new concept can be explicitly described in the following. In this centralized problem, delivery delay is not considered because it can amplify the discount effect on the cost of the manufacturer when the delivery delay is theoretically significant. Consequently, the NPV of the costs and the revenues of the manufacturer are negligible and have no impact on the optimization of the two-level plans. Thus, the delivery is not considered in this approach to avoid this unrealistic situation.

Theorem: A set of optimal production plans of the supplier and the manufacturer exists in $I_{i(t-1)} \times Q_{it} = 0$, $\forall t$, $i \in \{0, 1\}$, where I_{it} is the inventory of site i at the end of period t in the $2\text{ULS}_{\text{P(WCR)}}$ problem with associated assumptions in Section 1.

Proof: See Bian (2017) section 5.4.3.

Algorithm of the centralized approach

Figure 4 indicates that this approach has a Russian doll type procedure. This procedure is adopted by dynamically calculating the optimal (sub-)plan of the manufacturer every time we compute the arc value for the master problem at the supplier level in the recursion. Therefore, key elements are the formulations of the arc value of the

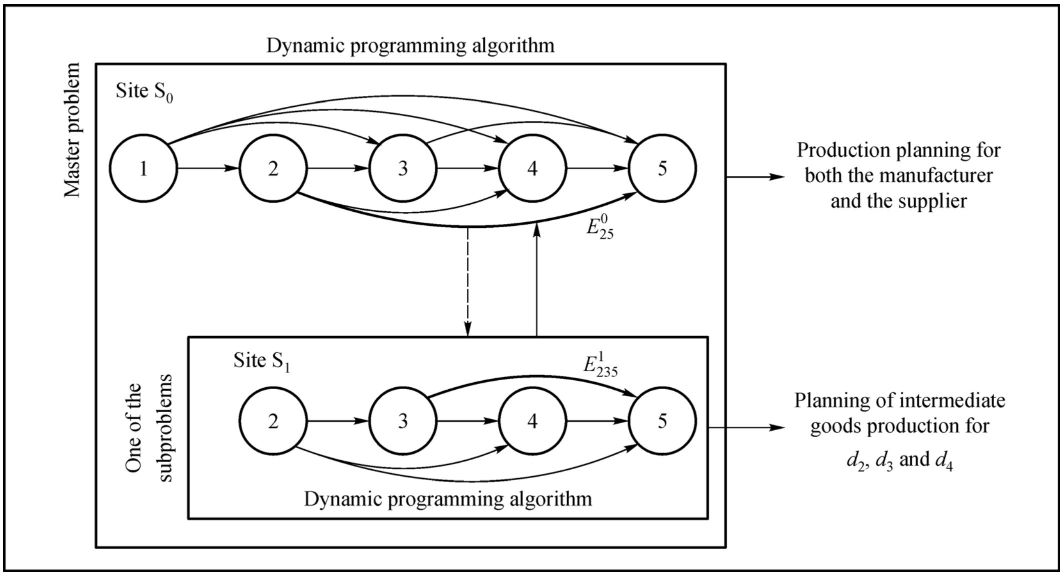

Fig. 4 Centralized approach for $2ULS_{P(WCR)}$.

sub-problem and master problem. The former allows us to integrate the interdependency among levels by including all costs and revenues that depend on the manufacturer's plan. When these arc values are calculated, we can directly apply the recursion of the single-level algorithm to obtain the optimal value of the sub-problem. The latter differs from those in the single-level problem because the optimal value of the corresponding sub-problem is considered.

- We denote that a sub-problem, associated to the E^0_{tk} of the master problem, is Sub_{tk};
- The arc value of the master problem between period t and k is E^0_{tk};
- For valuing E^0_{tk}, we must optimally resolve Sub_{tk}. The arc value between t' and k' in Sub_{tk} at level 1 is denoted by $E^1_{tkt'k'}$;
- The optimal value to reach node k' at level 1 from node t in Sub_{tk} is denoted by $Opt^1_{tkk'}$;
- We assume that all required intermediate goods are delivered at period t by the supplier.

First, in Sub_{tk}, the arc value of $E^1_{tkt'k'}$ consists of the inventory holding costs (denoted by $Inv^0_{tt'k'}$), all WCR financing costs ($WCR^0_{tkt'k'}$), the revenue ($R^0_{t'k'}$) of the supplier, and all logistics and financial costs of the manufacturer for the production of $d_{t'k'}$ in period t'. The four indices of $WCR^0_{tkt'k'}$ are all necessary, given that $WCR^0_{tkt'k'}$ presents the WCR of the supplier's logistic cost that depends on the production of $d_{t'k'}$. In addition, t and k are used to link this calculation to the associated sub-problem Sub_{tk}. Particularly, all products manufactured at period t by the supplier uniformly share the setup cost. With this unit setup cost, the WCR of setup cost is computed. The quantity of these products is found as d_{tk} in Eq. (22). Considering that we calculate $E^1_{tkt'k'}$, the intermediate goods to satisfy the external demand from $d_{t'}$ to $d_{k'-1}$ are already produced by the supplier at period t and can be delivered to the manufacturer at period t'. Furthermore, based on the ZIO property for the $2ULS_{P(WCR)}$ problem, the production quantity can only be the sum of demands in the following periods. Thus, the components of $E^1_{tkt'k'}$ can be written as follows:

$$Inv^0_{tt'k'} = \sum_{t=t'}^{t'-1}\sum_{q=t'}^{k'-1} h^0_{0l} d_q, \tag{21}$$

$$\begin{aligned} WCR^0_{tkt'k'} = {} & a_0 \cdot \sum_{q=t'}^{k'-1} d_q \left(\sum_{j=t+r_0}^{T} \frac{1}{(1+\alpha_0)^j} - \sum_{j=t'+r_1}^{T} \frac{1}{(1+\alpha_0)^j} \right) \\ & + p_0 \cdot \sum_{q=t'}^{k'-1} d_q \cdot \sum_{j=t}^{t'+r_1-1} \frac{1}{(1+\alpha_0)^j} \\ & + \frac{s_0}{d_{tk}} \cdot \sum_{q=t'}^{k'-1} d_q \cdot \sum_{j=t}^{t'+r_1-1} \frac{1}{(1+\alpha_0)^j} \\ & + h_0 \cdot \sum_{q=t'}^{k'-1} d_q \cdot \sum_{w=t}^{t'-1} \sum_{j=w}^{t'+r_1-1} \frac{1}{(1+\alpha_0)^j}, \end{aligned} \tag{22}$$

$$R^0_{t'k'} = v^0_{0(t'+r_1)} \sum_{l=t'}^{k'-1} d_l. \tag{23}$$

Revenue $R^0_{t'k'}$ only depends on the manufacturer's production plan to produce $d_{t'k'}$ at period t'. Thus, the supplier can receive the payment in period $t' + r_1$. Moreover, the two other components, such as the logistics and financial costs of the manufacturer, $LC^1_{t'k'}$ and $\beta_1 \times WCR^1_{t'k'}$, are directly modeled with the proposed WCR model in Section 3. To simplify the formula, we denote:

$$a^j_{it} = a_i \cdot \frac{1}{(1+\alpha_j)^t}, \tag{24}$$

and other cost parameters in the same way.

$$LC^1_{t'k'} = \left(a^1_{1(t'+r_1)} + p^1_{1t'}\right) \sum_{l=t'}^{k'-1} d_l + s^1_{1t'} + \sum_{l=t'}^{k'-1} d_l \sum_{q=t'}^{l-1} h^1_{1q}, \tag{25}$$

$$WCR^1_{t'k'} = \sum_{l=t'}^{k'-1} d_l \left[\left(\sum_{j=t'}^{T} a^1_{1(j+r_1)} - \sum_{j=l}^{T} a^1_{1(j+r_2)}\right) + \sum_{j=t'}^{l+r_2-1} p^1_{1j} + \sum_{j=t'}^{l+r_2-1} \frac{s^1_{1j}}{d_{t'k'}} + \sum_{q=t'}^{l-1} \sum_{j=q}^{l+r_2-1} h^1_{1j}\right]. \tag{26}$$

Combining the holding and financial costs of the supplier and all costs of the manufacturer, the formulation of $E^1_{tkt'k'}$ is written as follows:

$$E^1_{tkt'k'} = Inv^0_{tt'k'} + \beta_0 WCR^0_{tkt'k'} - R^0_{t'k'} + LC^1_{t'k'} + \beta_1 WCR^1_{t'k'}. \tag{27}$$

Using this formulation, we can compute the optimal value of sub-problem, $Opt^1_{tkk'}$, by adopting a recursion compared with the one in the single-level algorithm, which is:

$$Opt^1_{tkk'} = \min_{t' \in [t, k'-1]} \{Opt^1_{tkt'} + E^1_{tkt'k'}\}. \tag{28}$$

The values of Opt^1_{tkt} are set to zero. With this recursion, we sequentially compute the optimal value of $Opt^1_{tk(t+1)}$, then $Opt^1_{tk(t+2)}, \ldots,$ until Opt^1_{tkk}. Opt^1_{tkk} is the optimal value of Sub_{tk}, which is needed to determine the value of E^0_{tk} in the master problem.

Second, in the recursion of the master problem, we calculate the arc values at the supplier level. Thus, E^0_{tk} includes the costs that are independent of the optimal plan of the corresponding sub-problem (purchasing, setup, and production costs) and the optimal value of the corresponding sub-problem. Therefore, the arc value at supplier level is written as follows:

$$E^1_{tk} = \left(a'_{0(t+r_f)} + p'_{0t}\right) \sum_{l=t}^{k-1} d_l + s'_{0t} + Opt^1_{tkk}. \tag{29}$$

With all the elements mentioned above, the final recursion is formulated as below, where $Cost_t$ represents the minimal total cost of both sites minus the revenue of the supplier. Moreover, the beginning of the planning horizon is set to zero. Thus, $Cost_0 = 0$.

$$Cost_t = \min_{j \in [0, t-1]} \{Cost_j + E^0_{jt}\}. \tag{30}$$

To obtain the optimal value of the problem, we should sequentially compute $Cost_t$ from $t = 1$ to $t = \mathrm{T}$. The optimal value is $Cost_\mathrm{T}$, which can be obtained in $\mathrm{O}(\mathrm{T}^4)$. The corresponding pseudo-code is given in Algorithm 1.

Algorithm 1 Solving $2\mathrm{ULS}_{\mathrm{P(WCR)}}$

```
Require: All parameter values
  for k − 1 to T do
    for t = 0 to k − 1 do
      for l = t + 1 to k do
        for q = t to k − 1 do
          if Opt¹_tkl > Opt¹_tkq + E¹_tkql then
          Opt¹_tkl > Opt¹_tkq + E¹_tkql
          end if
        end for
      end for
      if Cost[k] > Cost[t] + E⁰_tk then
        Cost[k] = Cost[t] + E⁰_tk
      end if
    end for
  end for
```

5 Numerical tests for the $2\mathrm{ULS}_{\mathrm{P(WCR)}}$ model

5.1 Optimal production planning comparing both approaches

In this section, we show the differences between results (optimal production plans and different costs) obtained by the classic MLLP and $2\mathrm{ULS}_{\mathrm{P(WCR)}}$ models with sequential and centralized approaches. These differences illustrate the influence of considering the financing cost of OWCR on an optimal production program at both levels. In the following tests, the optimal program using the traditional MLLP model for two-level problem is denoted by $P^{i\mathrm{M}}_{\mathrm{opt}}$ (i = 0 for the supplier's plan, and i = 1 for the manufacturer's plan). Moreover, $P^{i\mathrm{S}}_{\mathrm{opt}}$ and $P^{i\mathrm{C}}_{\mathrm{opt}}$ represent the optimal programs calculated by $2\mathrm{ULS}_{\mathrm{P(WCR)}}$ model with the sequential and centralized approaches (of the supplier (with i = 0) and of the manufacturer (with i = 1)), respectively. The tests are organized to compare among $P_{\mathrm{opt}}^{i\mathrm{M}}$, $P_{\mathrm{opt}}^{i\mathrm{S}}$ and $P_{\mathrm{opt}}^{i\mathrm{C}}$ with the same set of parameter values provided to show the differences in the production programs. For the following tests, we adopt the demand of one instance of Dellaert and Jeunet (2000) (ph2in01st1de01, demand 1) over 24 periods. Table 3 indicates the values of other parameters (S stands for Supplier, and M stands for Manufacturer).

Table 3 Parameter values for the 24-period instance

Parameter	S	M	Parameter	S	M
v_i	35	50	h_i	1	2
p_i	3	3	α_i	0.05	0.01
s_i	800	600	β_i	0.03	0.03
a_i	3	35	$r_2 = r_1 = 2$, $r_0 = 1$		

We initially compare the optimal programs, which are separately calculated by the classic MLLP and $2\mathrm{ULS}_{\mathrm{P(WCR)}}$ models with sequential and centralized approaches. Figures 5 and 6 illustrate the differences among $P_{\mathrm{opt}}^{i\mathrm{M}}$, $P_{\mathrm{opt}}^{i\mathrm{S}}$ and $P_{\mathrm{opt}}^{i\mathrm{C}}$ (with i = 0, 1), respectively. As mentioned above, to compare with the same parameters considered in the classic MLLP model, we thus set $a_i = p_i = v_i = 0$ with i = 0, 1, and $r_0 = 0$ to focus on the setup and inventory holding costs. Moreover, given that the purchasing cost is not considered in test 1, the payment delay for raw material r_0 is thus irrelevant in this test. We also compare the total number of products held in the inventory of both levels. Table 4 indicates the number of setups. As a result, we observe the following:

• Based on the plans obtained by the MLLP model, we prefer to hold the finished products in the manufacturer instead of holding semi-finished products in the supplier. This finding is explained by the fact that the setup cost of the supplier is more expensive than that of the manufacturer, and the difference of unit holding cost between the two levels is relatively small. Quasi-synchronized plans of two levels are shown in Figs. 5 and 6, which allows the reduction of the inventory level in the supplier.

• Comparing the plans without and with the financial aspects, we consider that in $2\mathrm{ULS}_{\mathrm{P(WCR)}}$ problem, the number of setups is generally increased (from 5 to 6 at the supplier level and from 6 to 10 or 8 at the manufacturer level). This finding confirms the observation in the tests for

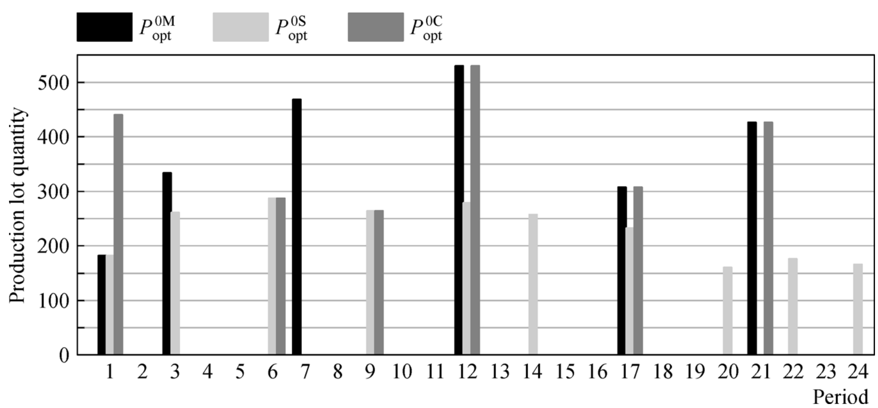

Fig. 5 Comparison of the optimal production programs of the manufacturer with different approaches.

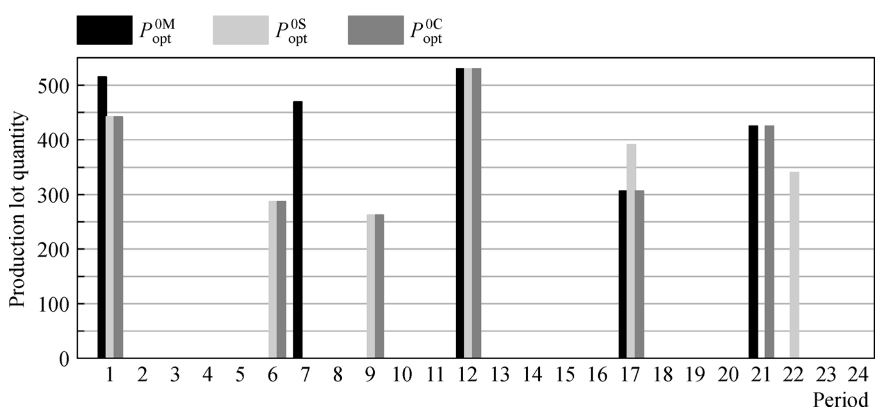

Fig. 6 Comparison of the optimal production programs of the supplier through different approaches.

the single-level problem, which indicates the interest to produce more frequently with a small lot size to reduce financial consequences.

• On the one hand, using the sequential approach can significantly reduce the manufacturer's inventory level for its prioritized optimization objective. On the other hand, the centralized approach balances the inventory levels between the two levels for a global optimization objective.

5.2 WCR comparison with varying delays in payment in a centralized case

In the following paragraphs, we test the influence of payment delays r_2 and r_1 on the two-level plans in a centralized case. Table 3 indicates the parameter values. We change β_0 and β_1 to 0.05. In each series of test, the considered delays r_1 and r_2 vary from 2 to 18 with a step of 8.

Figure 7 illustrates the setup numbers with varying r_1 and r_2 in the centralized approach. The results indicate that

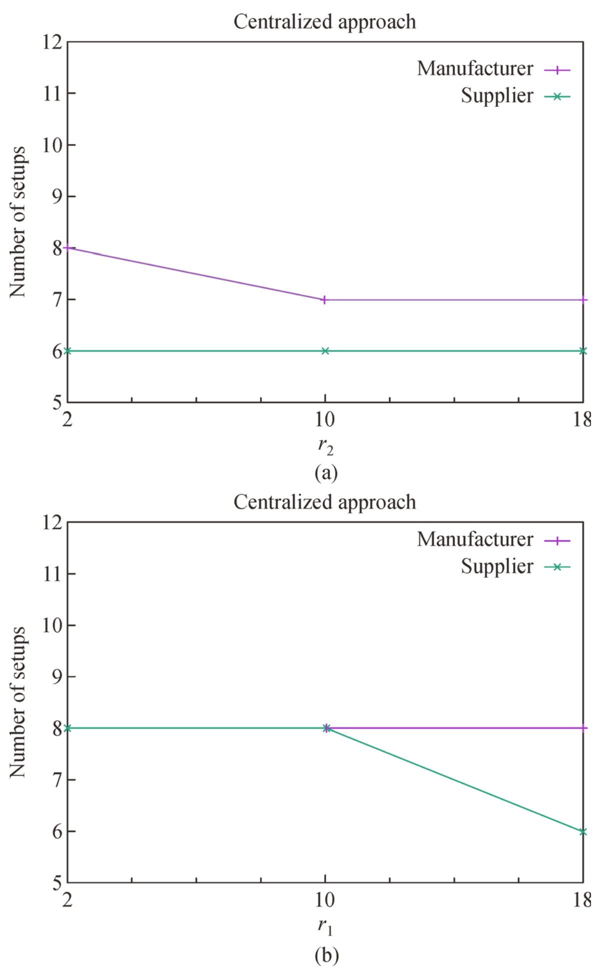

Fig. 7 Comparison of production plan with varying delays in payment in the centralized case.

the variation of delays in payment has an additional effect on downstream companies. The increase in supplier's profit is observed in the centralized approach. Moreover, varying r_1 results in an increased total cost of the manufacturer for the centralized objective. Table 5 reveals the significant increases of the objective value (OV) and the total cost of manufacturer (TC_1) with varying r_2 and the considerable decreases of these terms with varying r_1. Therefore, the strategies to improve OV are to prolong r_2 and reduce r_1 as much as possible (e.g., by adopting new technologies, such as digital supply chain and 5G).

Table 4 Comparison of the total products held in inventory over time and of the number of setups

Approaches	Supplier	Manufacturer
	Total products held (in unit/period)	
MLLP	670	3085
Sequential	1840	1230
Centralized	1005	2065
	Number of setups	
MLLP	5	6
Sequential	6	10
Centralized	6	8

Table 5 Consequences of varying delays in payment on financial terms

Varying parameter	ΔOV	ΔTC_1	$\Delta Profit_0$
r_2	59979.8	60742.7	762.9
r_1	-31035.1	-61904.4	-30869.3

6 Conclusions

In this study, we initially extend the previous single-level problem to a supplier–manufacturer two-level problem with a serial chain structure. After establishing the associated WCR model for this case, the corresponding mathematical model of this problem is developed, considering the WCR financial cost of the two levels. For the solution procedure, the sequential approach, which prioritizes the maximization of the manufacturer's profit than the supplier's, and the centralized approach, with a global profit maximization objective, are proposed. The sequential approach certainly consists a direct application of the single-level algorithm at two levels. In addition, the ZIO property remains valid with certain assumptions for the centralized approach. This property allows us to develop a revised dynamic programming-based algorithm, wherein the interdependency among levels is considered by the arc valuation. The observations obtained in the tests of the single-level problem are confirmed through the numerical tests. Furthermore, other observations are related to the interdependency among levels.

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