

Ka Fai NG, Hong K. LO, Yue HUAI

Robust public-private partnerships for joint railway and property development

© The Author(s) 2017. Published by Higher Education Press. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0>)

Abstract The involvement of the private sector in the construction or operation of an infrastructure project may enhance the financial viability of projects, which facilitates the formation of public-private partnership (PPP) for project delivery. PPP exploits the strength of the private sector by shifting certain project risks from the public party to the private sector who can efficiently manage certain risks. In joint railway and housing development, the approach of bundling railway and housing development (R&HD) allows cross-subsidization between immense railway construction cost and profitable housing rental revenue. This approach also provides flexibility in incorporating PPP models by distributing railway and housing revenues and costs and their inherent risks properly to the public and private sectors. Ng and Lo (2015a) developed an evaluation framework for joint railway and property development, which evaluates PPPs based on financial and construction criteria for selecting the best suitable PPP for a particular project. This study, which is based on the framework in Ng and Lo (2015a), aims to examine the robustness of various PPP configurations. This study analyzes the effects of PPP configurations on stakeholders' risks and returns under population or demand growth and railway construction cost uncertainties. The eventual outcome of particular PPP configurations is also examined. This study also seeks to answer the following questions: How would optimal configuration change under highly volatile population and railway construction cost? Are there PPP configurations that are robust to these uncertainties and those that are sensitive to a particular uncertainty? This understanding is critical for managing risks and facilitating the formation of appropriate PPP for R&HD.

Keywords public-private partnership, BFOOD, housing and railway development

1 Introduction

Public-private partnership (PPP) gained popularity in implementing infrastructure projects. PPP combines the strengths of the public and private sectors by properly assigning their role, responsibility, and risk in accordance to their capabilities in handling risks. Tang and Lo (2010) classified various PPP models in railway and property development project by defining the BFOOD nomenclature. BFOOD entails decisions to build, fund, own, operate, and develop property by the government (G), the railway sector (R), and the developer (D). For example, $B_R F_R O_R O_R D_{R/D}$ denotes a PPP model, wherein the railway sector builds, funds, owns, operates the railway, and shares property development above the station with a private housing developer. For simplicity, we assume that only one private company (R) may build, fund, own, operate the railway, and develop the property. BFOOD decisions in railway and property developments often involve massive investment and cooperation among stakeholders over a long period of time. Thus, quantitative land use and transport models must be developed to evaluate the performance of railway and housing projects in financial, transportation, and construction terms and provide guidance in selecting the appropriate PPP.

Researchers studied the application of PPP in transportation in a descriptive manner. Zhang (2005) identified five critical success factors of PPP projects through a survey; these factors are: (1) Favorable investment environment, (2) economic viability, (3) reliable concessionaire consortium with strong technical strength, (4) sound financial package, and (5) appropriate risk allocation via reliable contractual arrangements. Significance index and ranking can be applied to the critical success factors. Thus, the relative importance of those factors can be quantified.

Received August 23, 2017; accepted September 11, 2017

Ka Fai NG, Hong K. LO (✉), Yue HUAI

Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Hong Kong, China
E-mail: cehklo@ust.hk

Soomro and Zhang (2011) studied 35 transportation PPP cases worldwide and summarized specific failure reasons behind each PPP case in addition to fundamental financial and technical concerns. Regarding quantitative analysis on project management, real options analysis (Myers, 1977; Dixit and Pindyck, 1994; Trigeorgis, 1996) in financial engineering serves as an effective evaluation tool for analyzing the value of an option, which can be deferred when new information is received over time. Ho and Liu (2002) considered risks as major influence variables in a project; they adopted the real options approach to evaluate the financial viability of privatized infrastructure projects, such as airport terminals. Project value and construction cost are modeled as stochastic processes. Numerical solutions were also conducted. Ashuri et al. (2012) specified the evaluation of build-operate-transfer (BOT) highway projects and considered traffic demand as the only risk variable. Li et al. (2015) addressed transit investment decision by considering the timing and type of transit provision under population uncertainty in a real options framework.

The synergy between railway and housing developments was studied for decades using the static framework without incorporating the effects of risks. Housing developments near rail stations create new transport demands and increase railway ridership. Housing rents or the values of these housing developments benefit from the increased accessibility introduced by railway services. Railway revenue varies with the uncertain population demand, whereas railway construction cost varies with the public or private party responsible for the construction. Thus, railway profit is influenced by revenue and cost and their underlying uncertainties. Housing revenue varies with population housing demand and the housing affordability of households. The intertwined relationships among housing supply, population housing demand, and railway service can be captured by the combined bid-rent and residential choice model (Ma and Lo, 2012). The model describes to whom and at what price the developers rent out their properties (bid-rent process) and how households choose their housing and travel modes (residential choices). The interplay between these two perspectives defines the residential location choice equilibrium. Thus, the resultant rental value and railway passengers are captured.

Ng and Lo (2015a) proposed an evaluation framework for PPP under the combined bid-rent process and residential choice model; this model compared the performances of various PPP models in terms of housing and railway profits and the probability of project failure. They proposed that the risk variables, namely, demand, and railway construction cost, were modeled as stochastic processes. Geometric Brownian motion (GBM) is a typical stochastic process for modeling stock price behavior and is widely applied to various transportation issues on the demand side; these issues include the transit investment

problem (Gao and Driouchi, 2013) and urban congestion relief investment problem (Saphores and Boarnet, 2006) under population uncertainty and the network design problem under origin-destination demand uncertainty (Chow and Regan, 2011a, 2011b). This study applies the GBM to model demand and construction risks given growth rates and volatilities. The discrete time binomial lattice model (Cox et al., 1979; Hull, 2009) was then used to represent the possible discrete states of stochastic variables over time. The payoff of each state at a given time was calculated using the land use model developed in Ma and Lo (2012). The project payoff included railway profit, housing profit, and total consumer surplus. Simulation was used to generate sample paths through the binomial lattice, which depicts possible sequences of the project. Overall payoff was obtained by summing up payoff states along the sample path. Various PPP models were differentiated by growth rates and volatilities with or without minimum profit guarantee (MPG) and the total profit cap (TPC) mechanisms. Finally, the PPP models were evaluated by contrasting the distribution of the project payoff, such as railway and housing profits.

The present study, which is based on the PPP evaluation framework developed in Ng and Lo (2015a), examines the robustness of various PPP configurations. First, we analyze how stakeholders' risks and returns are affected by different PPP configurations and how preferred PPP configurations are eventually selected. Second, we study how optimal configuration is altered when the population and railway construction cost become highly uncertain. Various volatility parameters in GBM are used to describe the degrees of uncertainties. Changes in the distributions of stakeholder benefits (i.e., housing and railway profits and the total consumer surplus) in PPP models can be studied, and optimal PPP configurations can be compared from less uncertain cases to highly uncertain cases in a sensitivity analysis. Finally, probable PPP configurations that are robust or sensitive to uncertainty can be identified.

This study is structured as follows. Section 1 introduces PPP models according to their unique features. Section 2 proposes the evaluation framework. Section 3 provides numerical examples to illustrate the applicability of the framework. Section 4 concludes the study with the main findings.

1.1 BFOOD decisions

Build (B) decision is either assigned to the government or the private company, which bears distinctive railway construction cost uncertainty due to the different technology used in railway construction. Fund (F) decision is either assigned to the private company or the government. Own (O) decision is not captured in this model. Own decision reflects the possible transfer of ownership after a predefined PPP contract period. The remaining value and payment in an ownership transfer should also be

considered at the beginning. We assume that ownership does not change throughout the planning horizon. Either the private company or the government operates the railway. The government faces a higher railway operating cost than the private company due to the difference in technology. However, payback mechanisms should be carefully considered when the private company is assigned to run the railway. Railway profit is often insufficient in sustaining railway operation. Thus, subsidy to the private company would increase their willingness to commit to its operation. Housing construction costs are differentiated in the decision of developing housing due to private company expertise in the field. Two payback mechanisms are incorporated in addition to the five decisions. MPG ensures that the private operator of the railway makes a minimum profit regardless of uncertainties. If the government funds the project in the initial railway construction stage, payment may be later required from the private company. Payment is established as TPC. Various PPP configurations can be established together with the above payback mechanisms, either from the government to the private company or vice versa. The following show five typical PPPs range from the most privatized to purely public involvement.

1.2 Model 1— $B_R F_R O_R O_R D_R$

All decisions are assigned to the private company, who bears all the population demand and railway construction risks. However, the private company faces a lower uncertain railway construction cost and a lower housing development cost based on their technological advantages. The government in this PPP grants land for railway and housing developments and may acquire excess revenue if the railway operating profit is higher than a predetermined ceiling as defined by TPC.

1.3 Model 2— $B_R F_R O_R O_R D_G$

Similar to $B_R F_R O_R O_R D_R$, the government instead of the private company develops housing, thereby attracting more residents into the area. However, the cost of housing development on the government is expected to be higher than that in Model 1.

1.4 Model 3— $B_R F_G O_G O_R D_R$

Similar to $B_R F_R O_R O_R D_R$, the private company is involved in the decisions to build, operate, and develop the property. However, the government funds and owns the project and bears railway construction risk during the construction period. The synergy of this PPP model is that the government utilizes the technology of the private company to build the railway with housing, which results in low railway construction uncertainty and low housing development cost. The government directly provides financial

support to the railway project and may acquire excess revenue after railway construction is completed in the form of TPC.

1.5 Model 4— $B_G F_G O_G O_R D_G$

The government is the main party in this model, which is involved in decisions to build, fund, own, and develop the property. The government invites a private company to run the railway. The railway is built by the government. Construction process is anticipated to be less efficient than that of a private company. Therefore, the government faces a high construction risk. The private company is relatively passive in this PPP model given that railway revenue highly depends on the uncertainty on population demand. MPG may be provided by the government to maintain the financial viability of railway operation.

1.6 Model 5— $B_G F_G O_G O_G D_G$

This model shows that the government is committed to all the decisions in BFOOD. This model is not a PPP model by definition, but we presented it as a base case for comparison.

2 Evaluation framework

2.1 Modeling uncertainties

This study considers change in total population demand D and the railway construction cost K as the major uncertainties that affect project payoff. GBM captures the diffusion processes that are frequently used to model stock price behavior in financial engineering. The dynamics of the change in total population demand D is modeled by the following continuous-time stochastic process.

$$\frac{dD}{D} = \mu_D dt + \sigma_D dz, \quad (1)$$

where μ_D and σ_D are the mean rate and volatility of the change in total population growth, respectively, and dz is the standard increment of the Wiener process. Equation (1) states that risk variable D is expected to grow at a mean rate μ_D per unit time in the long run, as described in the first term, and fluctuate with some noise and variability, as shown in the second term. The second risk variable, namely, railway construction cost K , is modeled using the same stochastic process as

$$\frac{dK}{K} = \mu_K dt + \sigma_K dz, \quad (2)$$

where μ_K and σ_K are the mean rate and volatility of the railway construction cost, respectively, and dz is the standard increment of the Wiener process.

GBM enables us to track risk variables over time in a fine resolution, such as from day to day. A discrete time model is more desirable in land use planning model given that the changes are likely to vary from period to period in a coarse time scale, such as in months or even quarters of a year. The binomial lattice model proposed by Cox et al. (1979) is adopted to describe changes in the risk variable in each time step by representing the GBM in a discrete time scale. The GBM described in Eq. (1) can be regarded as the limiting case of the discrete time binomial model when the time step is smaller and smaller (Hull, 2009).

Given the risk variable at a particular time in the binomial lattice model, the risk variable can increase by a multiple of u with probability q or decrease by a multiple of d with probability $(1-q)$ after one step, where $u > 1$, $d < 1$, and $0 < q < 1$. Figure 1(a) shows three steps of the binomial lattice with one single risk variable. For a single risk variable, jump amplitudes u and d and jump probability q are given by the following equations (Cox et al., 1979; Hull, 2009):

$$u = \exp(\sigma\sqrt{\Delta t}), \quad d = \exp(-\sigma\sqrt{\Delta t}), \quad (3)$$

$$q = \frac{\exp(\mu\Delta t) - d}{u - d}, \quad (4)$$

where μ and σ are the mean rate and volatility of the risk variable, respectively, and Δt is the length of a time step.

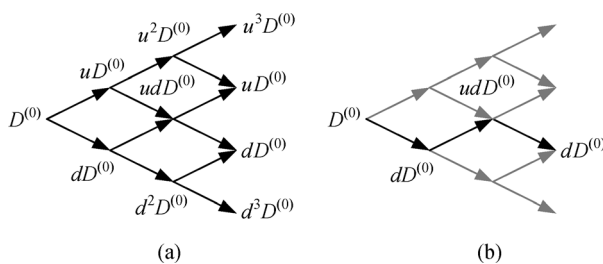


Fig. 1 (a) Three steps of the binomial lattice for population demand uncertainty; (b) a possible path

The two risk variables, namely, change in total demand D and railway construction cost K , are uncorrelated. Thus, we can extend the binomial lattice described above for one single risk variable to two uncorrelated binomial lattices with the corresponding jump amplitudes and jump probabilities. This assumption can be relaxed by applying the technique to transform two correlated variables into uncorrelated variables without difficulty (Hull, 2009). By considering the two risk variables D and K with the initial values at time $\tau = 0$ to be $D^{(0)}$ and $K^{(0)}$, the jump amplitudes and the jump probabilities are given by the following equations:

$$u_D = \exp(\sigma_D\sqrt{\Delta t}), \quad d_D = \exp(-\sigma_D\sqrt{\Delta t}), \quad (5)$$

$$q_D = \frac{\exp(\mu_D\Delta t) - d_D}{u_D - d_D}, \quad (6)$$

$$u_K = \exp(\sigma_K\sqrt{\Delta t}), \quad d_K = \exp(-\sigma_K\sqrt{\Delta t}), \quad (7)$$

$$q_K = \frac{\exp(\mu_K\Delta t) - d_K}{u_K - d_K}, \quad (8)$$

where u_D , d_D , u_K , and d_K are the jump amplitudes, q_D and q_K are the jump probabilities of the risk variables D and K , respectively. The length of a time step Δt is set at three months or a quarter. Let T be the total time of the project that includes housing and railway development. The railway construction cost appears only in the project construction phase, whereas total population demand is uncertain throughout the entire project time T . Railway construction starts at time T_0 and is completed at time T_K . N_D and N_K are the numbers of time steps in the binomial lattice for the risk variables $D^{(\tau)}$ and $K^{(\tau)}$, respectively. $N_D = T/\Delta t$, $N_K = T_K/\Delta t$ and $N_D \geq N_K$. We define time index $\tau = 1, 2, \dots, N_K, \dots, N_D$ as the month from the beginning of the project to the end of time T .

2.2 Monte Carlo simulation

Given the mean rates and volatilities of the risk variables, the binomial lattice of the total population demand D , and the railway construction cost K are formed based on the initial values. For any state at time τ , the values of risk variables $D^{(\tau)}$ and $K^{(\tau)}$ serve as the input for the combined bid-rent and residential choice model. The combined model evaluates the revenues and costs for housing development and railway operation at time τ . Monte Carlo simulation is applied to simulate the possible project sequences through the binomial lattice, as shown in Fig. 1(b). Overall payoff can be obtained by summing up the payoffs along the random path. Stakeholders' payoffs summed over the entire time period can be evaluated using the following equations. Given a random path along the binomial lattice model, the total railway cost, C_{rc} , can be expressed as

$$C_{rc} = \sum_{\tau=0}^{N_K} \frac{1}{(1+i)^\tau} K^{(\tau)}, \quad (9)$$

where i is interest rate. This term denotes the uncertainty of the railway construction cost during the railway construction period from $\tau = 0$ to N_K .

After railway construction, the operating profit of the railway can be expressed as

$$\begin{aligned} P_r &= \sum_{\tau=N_K+1}^{N_D} P_r^{(\tau)} \\ &= \sum_{\tau=N_K+1}^{N_D} \frac{1}{(1+i)^\tau} \left(R_r^{(\tau)}(D^{(\tau)}, K^{(\tau)}) - C_r^{(\tau)}(D^{(\tau)}, K^{(\tau)}) \right), \end{aligned} \quad (10)$$

where the first term inside the bracket is the railway revenue at time τ , and the second term is the railway operational cost at time τ . Railway revenue and the operating cost during the operational period are the functions of $D^{(\tau)}$ and $K^{(\tau)}$. Given $D^{(\tau)}$ and $K^{(\tau)}$, the railway revenue at time τ , $R_r^{(\tau)}(D^{(\tau)}, K^{(\tau)})$, and $C_r^{(\tau)}(D^{(\tau)}, K^{(\tau)})$ can be evaluated. The expressions of the railway revenue and cost are omitted for brevity. Readers may refer to the exact railway revenue and cost functions specified in the combined bid-rent and residential choice model in Ng and Lo (2017). From the housing developer perspective, the housing developer rental profit can be expressed as

$$P_h = \sum_{\tau=0}^{N_d} \frac{1}{(1+i)^\tau} \left(R_h^{(\tau)}(D^{(\tau)}, K^{(\tau)}) - C_h^{(\tau)}(D^{(\tau)}, K^{(\tau)}) \right), \quad (11)$$

where $R_h^{(\tau)}$ is the housing developer revenue at time τ , and $C_h^{(\tau)}$ is the housing development cost at time τ . $R_h^{(\tau)}$ and $C_h^{(\tau)}$ are functions of $D^{(\tau)}$ and $K^{(\tau)}$. Housing revenue and cost functions are skipped for brevity; they are defined in Ng and Lo (2017). A sufficiently large number of simulations are conducted to obtain the cumulative distribution function (CDF) of stakeholders' benefits. The distributions of railway and housing profits depict profit uncertainties due to the underlying population demand risk and railway construction risk. From the government perspective, the total consumer surpluses of residents sum over the entire project can be written as

$$CS_{\text{total}} = \sum_{\tau=0}^{N_d} \frac{1}{(1+i)^\tau} CS_T^{(\tau)}, \quad (12)$$

where $CS_T^{(\tau)}$ is the total consumer surplus at time τ defined in the combined bid-rent and residential choice model (Ng and Lo, 2017).

2.3 Risk sharing mechanisms: Minimum profit guarantee and total profit cap

MPG can be established to maintain the financial viability of a project, where the government provides a certain amount of revenue if the realized demand falls below a threshold of target demand. Minimum guarantee is a typical risk-sharing mechanism between the government and the private company in highway BOT projects (Ashuri et al., 2012), where payment is in the form of revenue. This study considers that the guarantee is measured in the form of railway profit, and payment is provided to the private company if the railway operating profit at any railway operational time period is lower than a predetermined profit margin, given that the private company suffers loss from the housing and railway development. MPG at time τ

is defined as

$$MPG^{(\tau)} = \begin{cases} \max \left(R_r^{(\tau)} \left(X_{MPG} - \frac{P_r^{(\tau)}}{R_r^{(\tau)}} \right), 0 \right), & P^{(\tau)} < 0 \\ 0, & P^{(\tau)} > 0 \end{cases}, \quad (13)$$

$$\tau = N_{k+1}, N_{k+2}, \dots, N_d,$$

where $P_r^{(\tau)}$ and $R_r^{(\tau)}$ are railway profit and revenue at time τ , respectively. If the total profit from housing and railway is smaller than zero, MPG is provided to subsidize the railway component. Railway profit margin is maintained at a predetermined level X_{MPG} , as shown inside the bracket of the first expression in the right-hand-side of Eq. (13). If the total profit from housing and railway is larger than zero, the government does not provide MPG to the private company, as shown in the second expression in the RHS of Eq. (13). The additional revenue from the MPG alters the railway profit and the performance of various PPP models. The total MPG sum over the entire project can be expressed as

$$MPG = \sum_{\tau=N_{k+1}}^{N_d} \frac{1}{(1+i)^\tau} MPG^{(\tau)}. \quad (14)$$

TPC allows the government to claim a proportion of the revenue if the private company profit margin at any railway operational time period is higher than a predetermined ceiling. Thus, TPC can be expressed as

$$TPC_{Y_{TPC}}^{(\tau)} = \max \left(R^{(\tau)} \left(\frac{P^{(\tau)}}{R^{(\tau)}} - Y_{TPC} \right), 0 \right), \quad (15)$$

$$\tau = N_{k+1}, N_{k+2}, \dots, N_d,$$

where Y_{TPC} is a predetermined maximum profit margin where the private company can gain. X_{MPG} and Y_{TPC} differ in various PPP models, as described in Section 2.4. The sum of the TPC over the entire project can be expressed as

$$TPC = \sum_{\tau=N_{k+1}}^{N_d} \frac{1}{(1+i)^\tau} TPC^{(\tau)}. \quad (16)$$

The expressions of the MPG and TPC are slightly different from the expressions used in highway BOT projects (Ashuri et al., 2012). The profit risk in a typical highway BOT project is positively related to traffic demand uncertainty. Therefore, a forecasted minimum demand defines the minimum revenue. However, in a railway and housing development project, the relationships among the population demand uncertainty, railway construction cost uncertainty with the railway profit, and the housing profit are rather sophisticated. For example, increasing population may pull down housing rent and housing revenue due to the housing supply effect, as proposed in Ng and Lo (2015b). Thus, the profit margin is

adopted as a relative profit control index for the private company, whereas other control indices could be used without modeling difficulty.

2.4 PPP models representation

The PPP models defined in Section 1 are differentiated by their unique characteristics, as presented via the above expressions. Parties are assigned to bear various risks in accordance with their ability to handle the risks and in the presence of payback mechanisms, namely, MPG and TPC. We present the differences among the PPP models in the following by utilizing various mean rates, volatilities, and cost coefficients in the binomial lattice model. In the following PPP models, the initial values of the population demand risk and railway construction cost risk at time $\tau = 0$ are the same and equal to $D^{(0)}$ and $K^{(0)}$, respectively. Railway operating costs during the operation period are assumed constant at $C_r^{(\tau)}(D^{(\tau)}, K^{(\tau)}) = C_r$. Housing development costs are assumed linear in $D^{(\tau)}$ with different unit development costs v_{h_1} across PPP models.

Table 1 summarizes the assignment of revenues and costs in the PPP models. The symbol $R \rightarrow G$ in the TPC and MPG columns denotes that money is paid by the private company to the government, and vice versa. Profit maximization is the main consideration from the perspective of private companies. The private company compares the PPP models in terms of the curves of their total profit, which includes housing, railway, and possibly MPG from the government. The expected and the standard deviation of the profit curves can be the performance indicators when comparing PPP models. Multiple criteria are involved from the government standpoint, which include consumer surplus and the private company profit. Table 2 shows the corresponding benefits of the private company and the government based on the assignment in Table 1.

The expected values of the benefits determine the financial value of PPP models. The risks encountered by the stakeholders dictate the performance of the PPP models. The higher the dispersion of the benefits is, the higher the risks are. Standard deviations and the coefficient of variation (COV) are applied to present the risk-reward ratio to the stakeholders. The proper balance between the benefit of the stakeholder and the risk encountered, as well as that between stakeholders, are crucial to overall

performance. The differences among PPP models are determined by the relative advantages of the stakeholder over the other. Private companies experience less uncertain railway construction cost and perform low housing construction and railway operation costs. The government develops housing that can attract more citizens into the area. The advantages are represented by the parameters assigned to the PPP models, as summarized in Table 3. v_{r_4} and v_{h_1} are the cost parameters of the railway operation cost and the housing development cost, respectively, as denoted in Ng and Lo (2017).

$\overline{\mu_D} \geq \mu_D$, $\overline{\sigma_D} \geq \sigma_D$, $\overline{\mu_K} \geq \mu_K$, $\overline{\sigma_K} \geq \sigma_K$, $\overline{v_{r_4}} \geq v_{r_4}$, $\overline{v_{h_1}} \geq v_{h_1}$, and $Y_2 \leq Y_1$. In Model 2, the government is responsible for developing housing, which attracts more residents into the area and performs high unit housing cost, as represented by a large mean of population demand, $\overline{\mu_D}$, and a high unit housing development cost parameter, $\overline{v_{h_1}}$.

3 Numerical examples

Two numerical examples using Monte Carlo simulation are presented below. The first studies the performances of PPP models under fixed uncertainties. The second simulation further considers the effects of population uncertainty to the performances of PPP models to identify possible robust PPP models. Both examples apply a simple network with one origin and one destination connected by an auto link and a railway link if it is constructed. Heterogeneous income classes are considered with high and low incomes and values of time. The parameters in this example are selected for illustration purposes.

3.1 Parameter setting

The parameters used in the model are listed below:

- Initial population demand: $D^{(0)} = 100$, initial railway construction cost: $K^{(0)} = 10^6$ HKD;
- Population demand uncertainty: $\mu_D = 0.075$, $\sigma_D = 0.2$, $\overline{\mu_D} = 0.15$, and $\overline{\sigma_D} = 0.2$;
- Railway construction cost uncertainty: $\mu_K = 0.03$, $\sigma_K = 0.1$, $\overline{\mu_K} = 0.045$, and $\overline{\sigma_K} = 0.1$;
- Railway operating cost: $v_{r_4} = 830$ HKD, and $\overline{v_{r_4}} = 1245$;
- Housing development cost for Model 1: $v_{h_1} = 7500$

Table 1 Assignment of BFOOD decision to parties with payback mechanisms

PPP models	Build (B)	Fund (F)	Own (O)	Operate (O)	Develop property (D)	MPG	TPC
B _R F _R O _R O _R D _R	R	R	R	R	R	/	R → G
B _R F _R O _R O _R D _G	R	R	R	R	G	G → R	R → G
B _R F _G O _G O _R D _R	R	G	G	R	R	/	R → G
B _G F _G O _G O _R D _G	G	G	G	R	G	G → R	/
B _G F _G O _G O _G D _G	G	G	G	G	G	/	/

Table 2 Benefits of private company and government

PPP models	Private company	Government
B _R F _R O _R O _R D _R	$P_h + P_r - C_{rc} - TPC_{Y_1}$	TPC_{Y_1}
B _R F _R O _R O _R D _G	$P_r - C_{rc} - TPC_{Y_1} + MPG_{X_1}$	$P_h + TPC_{Y_1} - MPG_{X_1}$
B _R F _G O _G O _R D _R	$P_h + P_r - TPC_{Y_2}$	$TPC_{Y_2} - C_{rc}$
B _G F _G O _G O _R D _G	$P_r + MPG_{X_1}$	$P_h - C_{rc} - MPG_{X_1}$
B _G F _G O _G O _G D _G	/	$P_h + P_r - C_{rc}$

HKD, and $\bar{v}_{h_1} = 11250$;

- f) MPG profit margin: $X_1 = 0.21$;
- g) TPC profit margin ceiling: $Y_1 = 0.66$, and $Y_2 = 0.55$;
- h) Entire project time: $T = 10$ years;
- i) Railway construction time: $T_K = 3$ years;
- j) Length of time step: $\Delta t = 1/4$ year = 3 months;
- k) Number of steps for the entire project: $N_D = T/\Delta t = 10/(1/4) = 40$;
- l) Number of steps for the railway construction: $N_K = T_K/\Delta t = 3/(1/4) = 12$;
- m) Number of days in one time period step: $n = 90$;
- n) Interest rate: $i = 0.04$;
- o) Simulation run: 500.

3.2 PPP comparisons under fixed uncertainties

Figures 2(a) and 2(b) plot the cumulative probability

Table 3 Parameter setting for PPP models in the evaluation framework

PPP models	μ_D	σ_D	μ_K	σ_K	v_{r_4}	v_{h_1}	X_{MPG}	Y_{TPC}
B _R F _R O _R O _R D _R	μ_D	σ_D	μ_K	σ_K	v_{r_4}	v_{h_1}	/	Y_1
B _R F _R O _R O _R D _G	$\bar{\mu}_D$	$\bar{\sigma}_D$	μ_K	σ_K	v_{r_4}	\bar{v}_{h_1}	X_1	Y_1
B _R F _G O _G O _R D _R	μ_D	σ_D	μ_K	σ_K	v_{r_4}	v_{h_1}	/	Y_2
B _G F _G O _G O _R D _G	$\bar{\mu}_D$	$\bar{\sigma}_D$	$\bar{\mu}_K$	$\bar{\sigma}_K$	v_{r_4}	\bar{v}_{h_1}	X_1	/
B _G F _G O _G O _G D _G	$\bar{\mu}_D$	$\bar{\sigma}_D$	$\bar{\mu}_K$	$\bar{\sigma}_K$	\bar{v}_{r_4}	\bar{v}_{h_1}	/	/

functions for the benefits of the private company and the government, respectively. Table 4 shows the corresponding expected values, standard deviations, and coefficients of variation. Figure 2(a) presents only four density curves. The private company is not involved and does not obtain benefits as in the last model.

Figures 2(a) and 2(b) show that the housing profit contributes the most benefit for either party if the parties are assigned to develop housing (i.e., high expected values for the private company in Models 1 and 3 and for the government in Models 2, 4, and 5), as also shown in Table 4. Figures 2(a) and 2(b) and Table 4 show that Models 1 and 2 maximize the individual benefit of the private company and the government, respectively. Considering the difference between Models 1 and 3, Model 1 presents a purely private company approach with only a fraction of TPC paid to the government. Model 3 shows that the private company bears the construction cost and pays TPC. Model 1 will always be the optimal model to maximize the benefit of the private company as long as the increase in TPC required to be paid to the government is compensated for the absence of the railway construction cost. Government benefit is maximized, as in Model 2, if the government is only responsible for housing development without funding, building, and operating the railway construction. Moreover, the maximum total consumer surplus is observed in Model 5 in Fig. 3 and Table 4. Total

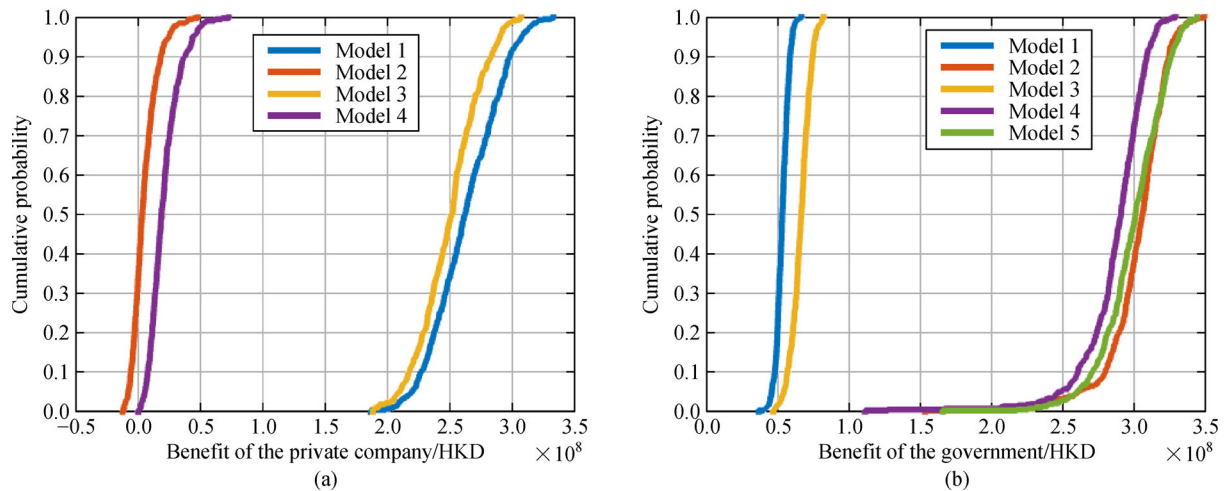
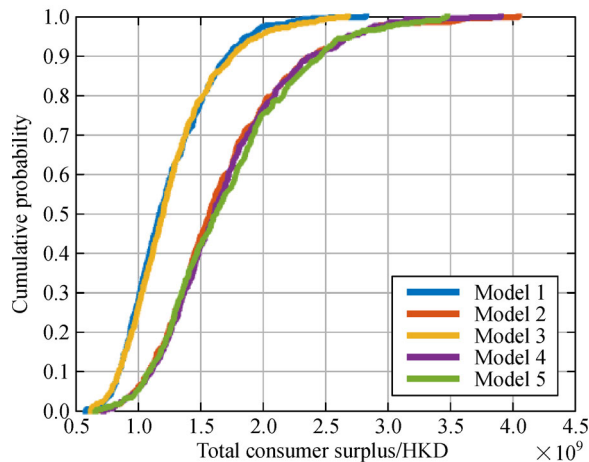
**Fig. 2** (a) Private company benefit; (b) government benefit in PPP models

Table 4 The resultant private company benefit, government benefit, and total consumer surplus

Model	Private company (R)			Government (G)			Consumer surplus		
	Expected value /(10 ⁷ HKD)	Standard deviation /(10 ⁷ HKD)	COV /%	Expected value /(10 ⁷ HKD)	Standard deviation /(10 ⁷ HKD)	COV /%	Expected value /(10 ⁹ HKD)	Standard deviation /(10 ⁸ HKD)	COV /%
1	26.300	2.740	10.43	5.330	0.434	8.14	1.240	3.500	28.16
2	0.520	0.986	189.73	30.200	2.200	7.29	1.670	5.380	32.13
3	25.000	2.420	9.70	6.610	0.672	10.15	1.260	3.670	29.20
4	2.110	1.190	56.41	28.700	2.280	7.96	1.690	5.200	30.76
5	–	–	–	29.900	2.260	7.55	1.710	5.390	31.60

consumer surplus is directly related to the total number of residents that move to the area for the entire planning horizon. More citizens are attracted to the area given that the government is assigned to develop housing in Model 5. Therefore, this approach produces the highest total consumer surplus.

**Fig. 3** Total consumer surplus in PPP models

As shown in Table 4, Models 2 and 4 produce a relatively high COV for the private company (i.e., 189.73% and 56.41%). The COV is minimized if the parties are assigned to decisions with resulting similar order of risks and benefits. For example, in Model 2, the private company build, fund, and operate the railway, where the major risk is contributed by railway construction and operation. The disproportional risk and benefit drastically increase the COV for the private company. Compared with Model 1, where the private company develops housing, the massive housing profit compensates the disproportionate railway construction and operation risk in Model 2. Therefore, the housing profit minimizes the COV for the private company. The same explanation applies to Model 4. The highly dispersed railway operation profit is only borne by the private company without a stable housing profit. The risk and benefit ratio is unequal in magnitude, which results in a high value of COV. As far

as minimizing COV is concerned, the joint assignment of developing housing and operating railway to the same party proportionally allocates risks and benefits to all parties.

To summarize the performances of the PPP models, Models 1 and 2 produce the maximum expected benefits to the private company and the government, respectively. By contrast, Models 1, 3, and 5 minimize the risk and benefit ratios for the two parties. Model 1 would be a good PPP model for maximizing the private company benefit and minimizing risks.

3.3 Sensitivity analysis on population demand

The performances of PPP models are further investigated under the changing population demand uncertainty. With the same parameter setting as in the previous section, the expected value and COV of the benefits of the private company and the government are analyzed when the mean rate μ_D increases. Four demand cases are considered. The mean rate of the population increases from Cases 1 to 4. First, we study the variations in housing profit and the railway profit. Figures 4(a) and 4(b) plot the cumulative probability density functions of the housing revenues and costs in Models 1 and 2, respectively, under different population demand mean rates. Figures 5(a) and 5(b) plot the corresponding cumulative probability density functions of the housing profits in Models 1 and 2. PPP Models 1 and 2 are selected as representative models given that these two models represent different cases for property development. The private company (Models 1, 3, and 5) or the government (Models 2 and 4) is assigned to the decision to develop property, which shows the differences in attracting residents in the area (i.e., different mean rates for uncertain demand, μ_D).

Figure 4(a) shows that housing revenue shifts more to the right than the housing development cost when the mean rate of the population demand increases. Result under the private company housing development shows that a high mean rate of the population leads to a high housing profit in Model 1, as shown in Fig. 5(a). The variation of the housing revenue also exhibits a magnitude larger than the housing cost when the mean of the population increases.

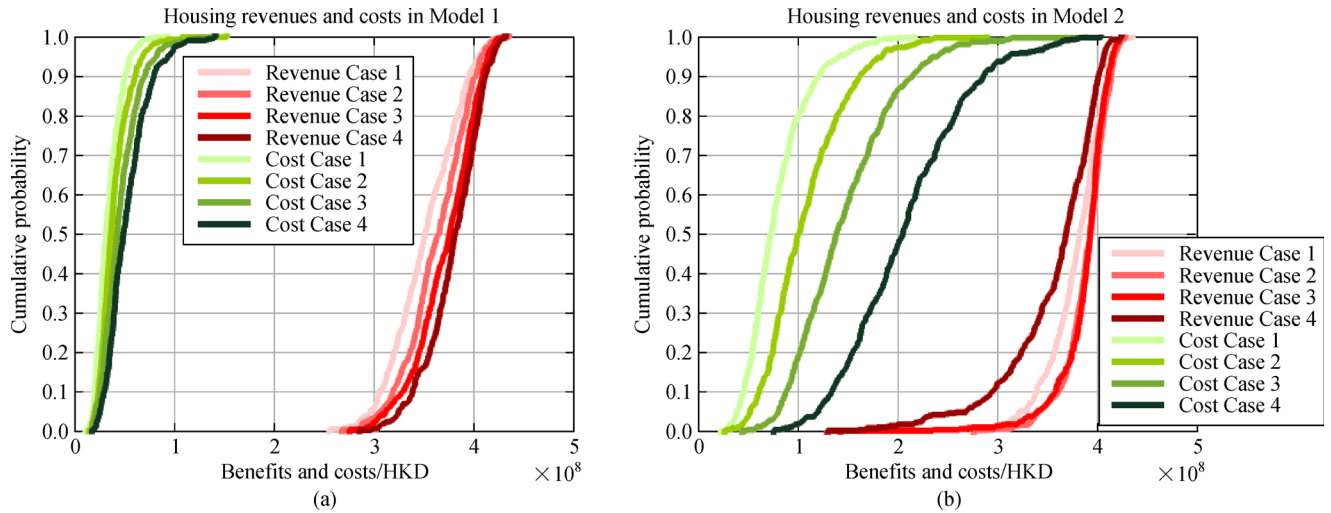


Fig. 4 Housing revenues and costs in (a) Model 1 and (b) Model 2

Thus, the profit curve in Fig. 5(a) follows similar variations as in the housing revenue curve in Fig. 4(a).

Figure 4(b) for Model 2 shows that the housing cost varies extensively when the mean rate increases compared with the housing revenue. Model 2 shows that the government develops housing with a high mean rate of the population. Housing revenue starts to drop when the mean rate increases to a high level. Overpopulation dramatically pulls down the housing rent through the supply effect, as presented in Ng and Lo (2015b); this effect compensates for the increase in housing supply due to increase in population; therefore, the total housing revenue and its variation decreases. Housing development cost increases with the increase in the number of housing supply. The profit curve in Fig. 5(b) follows a similar variation to the negative housing cost.

Railway revenues against the mean rate of population demand in Models 1 and 2 are plotted in Figs. 6(a) and

6(b), respectively. Increasing trends are seen for the expected value and the variation of the railway revenue when the mean rate of the population demand increases. The railway link constructed outperforms the auto link with congestion. Thus, the railway serves as a superior transport mode for the residents. When population demand increases to a high level, the railway attracts more residents. Therefore, the expected value of the railway revenue increases and variation is magnified. Railway revenue is higher in Model 2 than that in Model 1 because population demand is high in Model 2 when the government is assigned to develop housing. Given the fixed railway operation cost and fixed mean rate of railway construction cost, the railway profit curves can be subsequently represented by the revenue curves of the railway.

The results of the housing revenues, costs, and profits, as well as the railway revenues, are studied under the

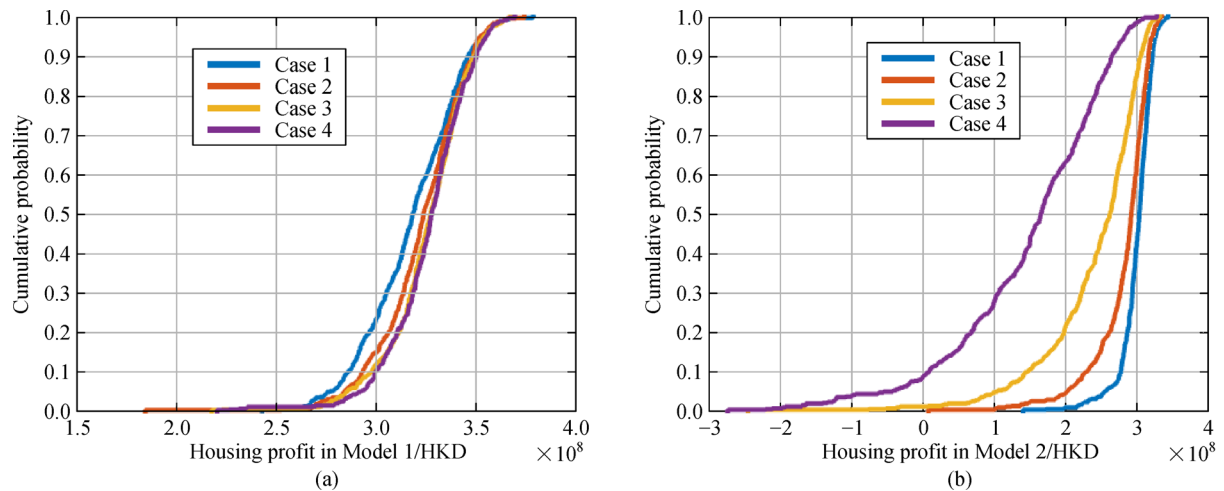


Fig. 5 Housing profit in (a) Model 1 and (b) Model 2

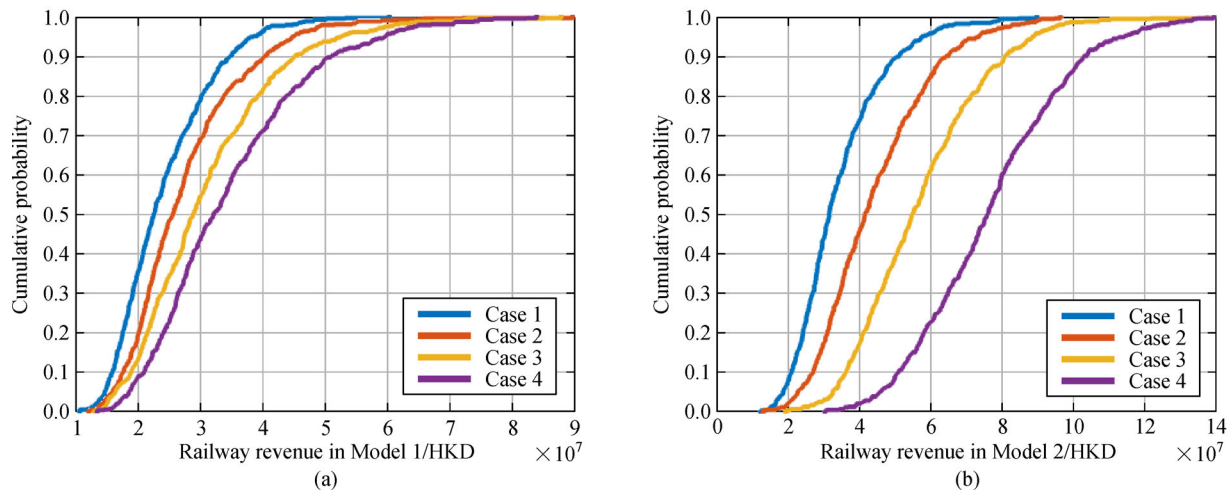


Fig. 6 Railway revenue in (a) Model 1 and (b) Model 2

increasing mean rate of population demand. The performance of PPP models is studied case-by-case. Table 5 summarizes the expected values, whereas Table 6 shows the COV for the benefits of the private company and the government for all PPP models under different population demand cases.

Figures 7(a) and 7(b) plot the benefits of the private company and the government, respectively. Figure 7(a) shows that the benefit curves of the private company in Model 1 are observed to follow similar patterns in the housing profit curves in Fig. 5(a). Model 1 presents that the private company is responsible for all the decisions in railway and housing. Therefore, the benefit is mainly contributed by the housing profit. An increasing trend is also observed in the railway profit curve in Model 1, as presented in Fig. 6(a). This finding adds to the increasing trend toward the overall benefit curves. Therefore, the benefit of the private company curves exhibits a similar pattern as in the housing profit curves. Figure 7(b) plots the

benefit of the government in Model 1, which is contributed by the TPC collected from the private company. When the mean rate of the demand increases, the expected value decreases, whereas the variation increases. Thus, the COV of the government increases, as shown in Tables 5 and 6. The COV for private company decreases and that for the government increases when the mean rate of the population increases. Therefore, the risk incurred by the increase in population is transferred from the private company to the government under the fixed profit margin as established in the TPC mechanism. Profit margin requires renegotiation when the population becomes substantial.

Model 2 indicates that the private company is in the position to build, fund, own, and operate the railway without massive housing profit to sustain its benefit. Figure 8(a) shows that the benefit curve for the private company shifts to the right when the mean rate increases. Figure 9(a) plots MPG with the increasing mean rates of population demand. The amount of MPG decreases when

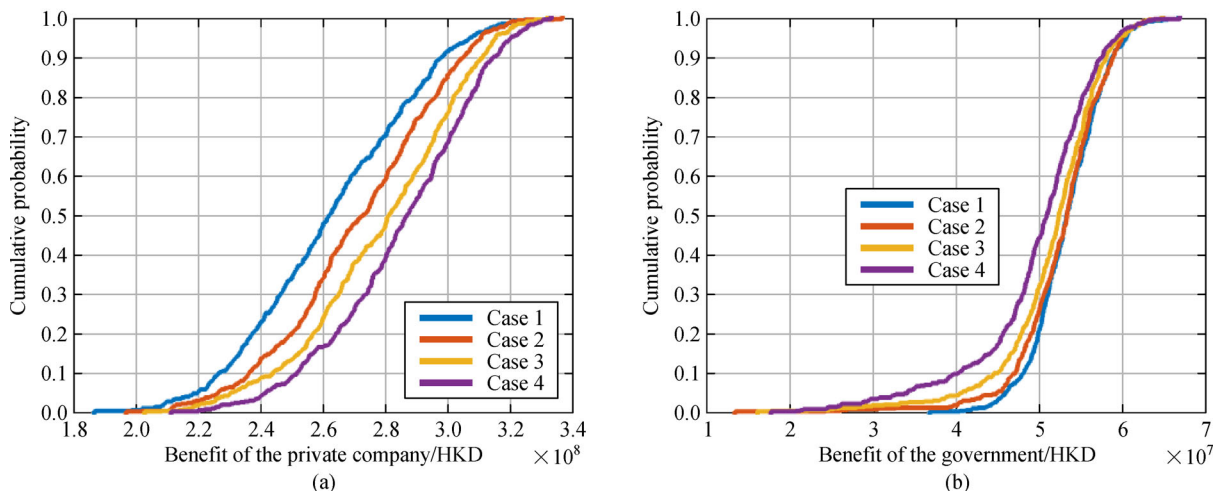


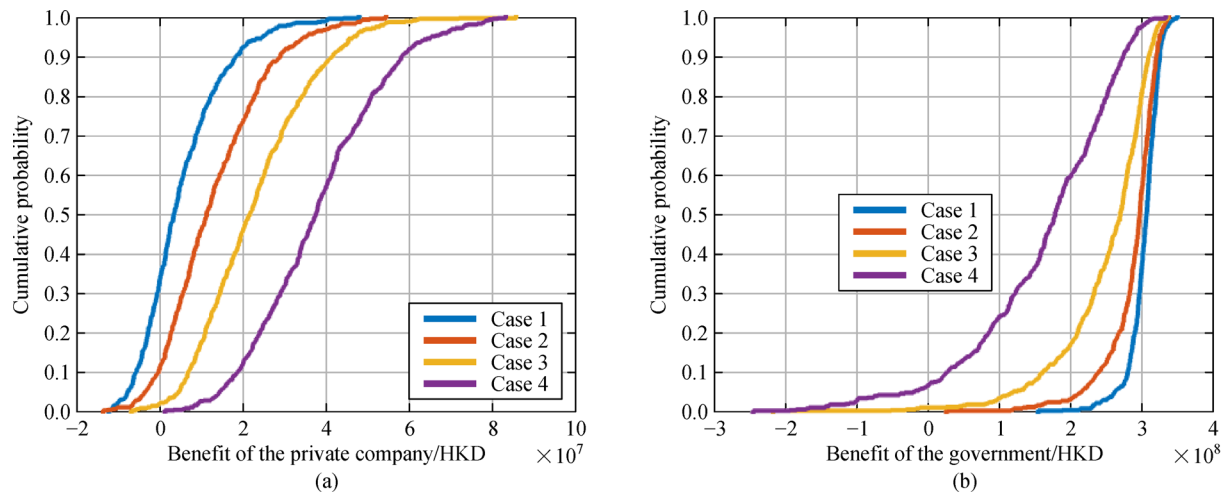
Fig. 7 Benefits of (a) the private company and (b) the government in Model 1

Table 5 Expected benefits of the private company (R) and the government (G) under different demand cases unit: 10^7 HKD

Case	Model 1		Model 2		Model 3		Model 4		Model 5	
	R	G	R	G	R	G	R	G	R	G
Case 1	26.25	5.33	0.52	30.22	24.97	6.61	2.11	28.67	0	29.92
Case 2	27.13	5.26	1.30	28.62	25.53	6.63	3.09	26.66	0	29.14
Case 3	27.87	5.15	2.29	24.86	26.31	6.68	4.39	22.30	0	25.92
Case 4	28.49	4.96	3.81	16.11	27.07	6.54	6.49	12.87	0	18.50

Table 6 Coefficients of variation of the benefits of the private company (R) and the government (G) under different demand cases unit: %

Case	Model 1		Model 2		Model 3		Model 4		Model 5	
	R	G	R	G	R	G	R	G	R	G
Case 1	10.4	8.1	189.7	7.3	9.7	10.2	56.4	8.0	–	7.5
Case 2	9.5	10.4	92.0	13.2	8.9	10.4	49.7	16.5	–	10.9
Case 3	9.2	12.4	59.1	26.5	8.4	10.3	42.2	33.4	–	22.9
Case 4	8.5	15.2	40.8	62.8	7.4	13.5	32.7	86.6	–	51.1

**Fig. 8** Benefits of (a) the private company and (b) the government in Model 2

population demand increases because the increasing population demand promotes the use of railway service, as explained in Fig. 6(a). Thus, the railway profit increases and the subsidy required (MPG) to maintain the financial sustainability is reduced. Fig. 9(b) shows that an increase in demand also induces a high TPC for the private company to the government. The overall effects of MPG and TPC produce the benefit curves of the private company in Fig. 8(a). The benefit curves in Fig. 8(b) for the government follow exactly the same in Fig. 5(b) due to substantial housing profit.

The disproportionate risk to benefit ratio borne by the private company undermines the performance of the model due to the configuration of this PPP model. The COV for the private company shown in Table 6 is extremely high compared with other models. Another finding is that the

COV decreases for the private company and increases for the government. When the mean rate of population demand increases, the expected value and the variation of the benefit for private company increase simultaneously although the ratio drops overall.

The difference between Models 3 and 1 is that railway construction cost is paid by the government. A tight TPC is created in return. Figures 10(a) and 10(b) plot the benefits of the private company and the government in Model 3, which follow similar patterns as Figs. 7(a) and 7(b). TPC payment from the private company reduces its benefit, whereas the COV of the company in Table 6 is smaller than that of Model 1. The tradeoff between the benefit (Model 1) and the risk and return ratio (Model 3) is subjected to the level of profit margin embedded in the TPC mechanism and the risk attitude of the private

company.

Model 4 shows the condition when the private company is only assigned to the railway operation without TPC and railway construction cost. Figures 11(a) and 11(b) plot the benefits for the private company and the government in Model 4, respectively. The benefit curves for the private company and the government show similar patterns as in Model 2. Railway construction cost is paid by the government in this model. The benefit of the private company is higher than that in Model 2 with a lower COV compared with Model 2 for any demand case. A comparison of Models 2 and 4 shows that the value of the TPC and the railway construction cost are crucial in assigning funding decision. The value of TPC should balance the benefits of the stakeholders and the risk and return ratios.

Model 5 serves as a base case where all decisions are assigned to the government given that the private company

has no benefit. Figure 12 shows that the benefit curve of the government shifts to the left and is spread over when the mean rate increases, which shows the decrease in benefit and increase in variation. The benefit curves follow the curves in Fig. 5(b). As the population increases to an extremely high level, the housing rent decreases because of the housing supply effect. The decrease in housing rent compensates the increase in housing supply due to the incorporation of the population, which pulls down the housing profit overall and the benefit of the government. Moreover, the model shows a small value of COV under the low demand case, whereas the COV increases dramatically with the mean rate. This finding indicates that the performance of Model 5 is sensitive to the increase in population demand.

All PPP models are compared with the expected values and the risk to return ratios, as shown in Tables 5 and 6. Model 1 produces the largest expected benefit for the

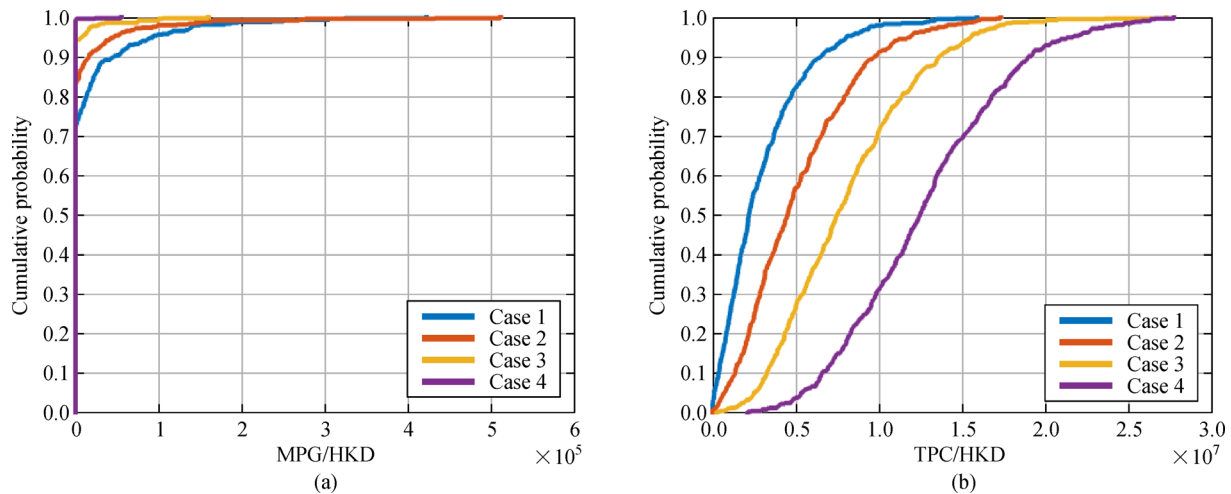


Fig. 9 (a) MPG and (b) TPC in Model 2

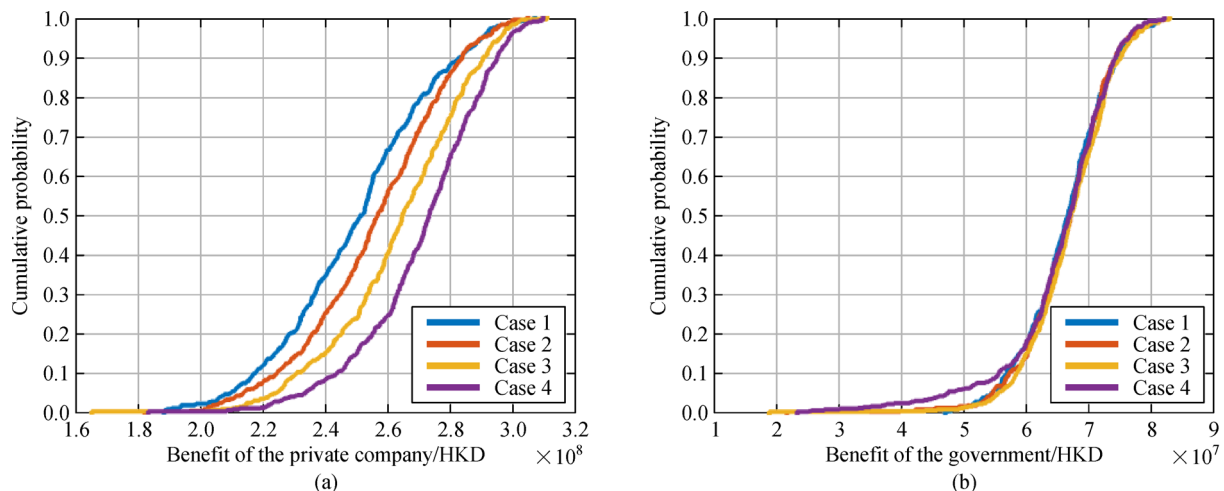


Fig. 10 Benefits of (a) the private company and (b) the government in Model 3

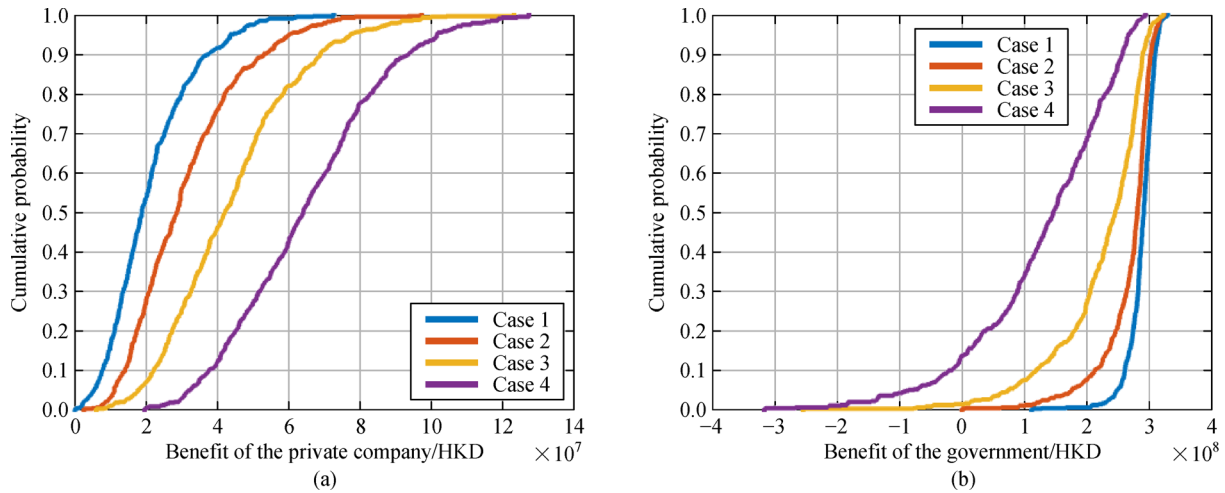


Fig. 11 Benefits of (a) the private company and (b) the government in Model 4

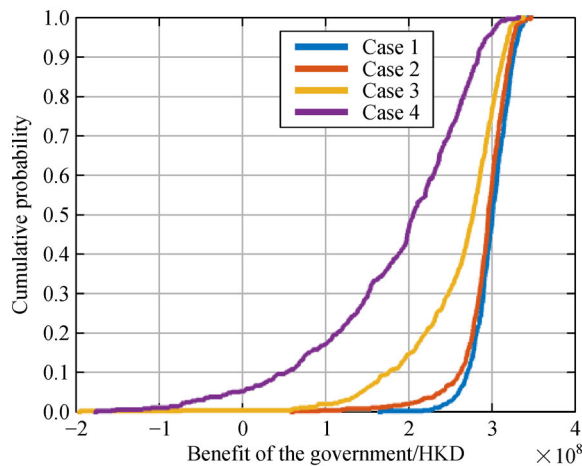


Fig. 12 Benefit of the government in Model 5

private company. The largest expected benefit for the government changes from Model 2 to Model 5 when the population increases. Models 1 and 3 perform fractional changes in COV for both parties, which demonstrate the robustness of the model under population uncertainty.

In summary, housing profit contributes most of the stakeholders' benefits. These stakeholders are assigned to develop housing (i.e., Models 1 and 3 for the private company, whereas Models 2, 4, and 5 for the government). Total consumer surplus is maximized when the government is assigned to develop housing given that more residents are attracted. Second, the joint decision between housing development and railway construction is always better in terms of minimizing the COV for all parties (i.e., Models 1, 3, and 5). This assignment allocates the proportional risks to the parties according to the magnitude of their benefits. Third, the establishment of TPC requires extensive numerical analysis to ensure that the railway construction cost is always or likely to be paid back by the

private company. The TPC also acts as a tool to balance stakeholders' benefits and the risk and return ratios. Lastly, the model is identified with the largest benefit and the lowest COV, but performing sensitivity analysis is important because vulnerable models, such as Model 5, are identified. This finding shows a low COV under low demand case, but is sensitive to increasing population due to the effect of overpopulation.

4 Conclusions

We adopted the evaluation framework in Ng and Lo (2015a) to assess the performance of various PPP models for railway and housing development projects under demand population and railway construction cost uncertainties; our analysis is based on the BFOOD configuration, as defined by Tang and Lo (2010). The evaluation framework rigorously expressed the roles and responsibilities for the private company and the government with payback mechanisms. This study also conducted probability analysis on population demand and railway construction cost uncertainties. Numerical examples demonstrated the applicability of the evaluation framework, from single uncertainty to varying uncertainty with increasing mean rate of population. The performances of the PPP models were analyzed through the benefits and the risk to return ratios, as measured by the expected value and the COV. Robust PPP models were identified based on the sensitivity analysis on the population demand. This study provided an evaluation framework and selection criteria for choosing PPP models in a railway and property development under uncertainties.

Acknowledgements The study was supported by the General Research Funds of the Research Grants Council of the HKSAR Government (Grant Nos. 616113 and 16222216).

References

- Ashuri B, Kashani H, Molenaar K R, Lee S, Lu J (2012). Risk-neutral pricing approach for evaluating BOT highway projects with government minimum revenue guarantee options. *Journal of Construction Engineering and Management*, 138(4): 545–557
- Chow J Y J, Regan A C (2011a). Real option pricing of network design investments. *Transportation Science*, 45(1): 50–63
- Chow J Y J, Regan A C (2011b). Network-based real option models. *Transportation Research Part B: Methodological*, 45(4): 682–695
- Cox J C, Ross S A, Rubinstein M (1979). Option pricing: A simplified approach. *Journal of Financial Economics*, 7(3): 229–263
- Dixit A K, Pindyck R S (1994). *Investment Under Uncertainty*. America: Princeton University Press
- Gao Y, Driouchi T (2013). Incorporating Knightian uncertainty into real options analysis: Using multiple-priors in the case of rail transit investment. *Transportation Research Part B: Methodological*, 55: 23–40
- Ho S P, Liu L Y (2002). An option pricing-based model for evaluating the financial viability of privatized infrastructure projects. *Construction Management and Economics*, 20(2): 143–156
- Hull J (2009). *Options, Futures and Other Derivatives*. 7th ed. New Jersey: Pearson/Prentice Hall
- Li Z C, Guo Q W, Lam W H K, Wong S C (2015). Transit technology investment and selection under urban population volatility: A real option perspective. *Transportation Research Part B: Methodological*, 78: 318–340
- Ma X, Lo H K (2012). Modeling transport management and land use over time. *Transportation Research Part B: Methodological*, 46(6): 687–709
- Myers S C (1977). Determinants of corporate borrowing. *Journal of Financial Economics*, 5(2): 147–175
- Ng K F, Lo H K (2015a). An option pricing-based evaluation of public-private partnerships for joint railway and property development. In: *Proceedings of the 20th Conference of Hong Kong Society for Transportation Studies (HKSTS)*
- Ng K F, Lo H K (2015b). Optimal housing supply in a bid-rent equilibrium framework. *Transportation Research Part B: Methodological*, 74: 62–78
- Ng K F, Lo H K (2017). On joint railway and housing development: Housing-led versus railway-led schemes. *Transportation Research Part B: Methodological* (in press)
- Saphores J D M, Boarnet M G (2006). Uncertainty and the timing of an urban congestion relief investment. *Journal of Urban Economics*, 59(2): 189–208
- Soomro M A, Zhang X (2011). Analytical review on transportation public private partnerships failures. *International Journal of Sustainable Construction Engineering and Technology*, 2(2): 62–80
- Tang S, Lo H K (2010). Assessment of public private partnership models for mass rail transit-an influence diagram approach. *Public Transport (Berlin)*, 2(1–2): 111–134
- Trigeorgis L (1996). *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. Cambridge: MIT Press
- Zhang X Q (2005). Critical success factors for public-private partnerships in infrastructure development. *Journal of Construction Engineering and Management*, 131(1): 3–14