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# Perspectives in multilevel decision-making in the process industry

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**Abstract** Decisions in supply chains are hierarchically organized. Strategic decisions involve the long-term planning of the structure of the supply chain network. Tactical decisions are mid-term plans to allocate the production and distribution of materials, while operational decisions are related to the daily planning of the execution of manufacturing operations. These planning processes are conducted independently with minimal exchange of information between them. Achieving a better coordination between these processes allows companies to capture benefits that are currently out of their reach and improve the communication among their functional areas. We propose a network representation for the multilevel decision structure and analyze the components that are involved in finding integrated solutions that maximize the sum of the benefits of all nodes of the decision network. Although such task is very challenging, significant research progress has been made in each component of this structure. An overview of strategic models, mid-term planning models, and scheduling models is presented to address the solution of each node in the decision network. Coordination mechanisms for converging the integrated solutions are also analyzed, including solving large-scale models, multiobjective optimization, bi-level programming, and decomposition. We conclude by summarizing the challenges that hinder the full integration of multilevel decision making in supply chain management.

**Keywords** supply chain optimization, enterprise-wide optimization, multilevel optimization, planning, scheduling

## 1 Introduction

The supply chains in the 21st century are highly globalized. Nowadays, having a product that has been manufactured 10,000 kilometers away delivered in two weeks seems very natural. With the information revolution, people have become used to receiving their orders almost instantaneously. However, such practice has placed tremendous pressure on supply chains by increasing their complexity and requiring them to be very responsive. In this scenario, making effective decisions in a timely manner is nearly impossible without a good decision support system.

The process industry serves as a good example of such complexity. A typical chemical company has suppliers distributed across different geographical locations, dozens of manufacturing sites, and customers all over the world. Decisions in the process industry must be made in consideration of the material flows throughout the supply chain together with the decisions related to the manufacturing process, including batch sizing and timing, defining production rates, parameter setting, and control. Process Systems Engineering (PSE) addresses the challenge of optimizing industrial processes with all its complexity (Sargent, 2005). The challenges in integrating R&D, manufacturing, and distribution functions have been recognized in the area of Enterprise-wide Optimization (EWO) (Grossmann, 2005). Several authors have attempted to identify the main challenges and opportunities involved in EWO. Shah (2005) thoroughly describes the challenges in the process industry supply chain and identifies the lack of integration of design and operational decisions as one of the main challenges, while Papageorgiou (2009) reviews the relevant literature on the modeling aspects that has been published before 2008.

A supply chain can be defined as a sequence of steps involved in the manufacturing and distribution of a product. This definition, which illustrates a horizontal process starting from the supply of raw materials to the

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production of finished goods (Fig. 1), can help one understand the flow of materials. However, the flow of information is not fully captured by this representation. Information, especially decisions, flow from the strategic level to the operational level, passing through the tactical level. In order to make optimal decisions, feedback must also flow in the opposite direction.

All decision levels are interconnected. For example, the facility location decisions that are made at the strategic level can affect the capacity for the tactical plan, which defines inventory targets for the scheduling of operations. Therefore, a truly optimal solution is one that yields the best possible value for the objective function (whether maximizing or minimizing) while considering the effects on all decision levels. Even though the availability of information inside enterprises has been improved by adopting enterprise resource planning (ERP) systems, planning processes are still being executed independently with minimal communication among different decision levels. Although coordinating various decision-making processes is a very challenging task, significant progress has been made in studying the components of this problem. In this paper, we present an overview of the elements involved in solving such problem. Our goal is to set the stage to accomplish the integrated optimization of all decision levels across a supply chain.

The decisions in supply chains are hierarchically organized and can be divided into strategic, tactical, and operational levels. The distinction between each level is not absolute and varies across companies. The strategic level includes long-term planning decisions that affect the structure of the supply chain network, the tactical level includes mid-term decisions related to the allocation and distribution of materials between manufacturing and storage units (warehouses and distribution centers), and the operational level includes decisions relating to the execution of manufacturing operations. Given that the planning horizons for each level are different, the timescales employed when modeling each decision level are also different. Figure 2 shows the general characteristics of the hierarchy of decision levels. Below the operational level there is a control layer. The integration of operational decisions and control also warrants further research. However, the decisions beyond the operational level will not be covered in this paper. For additional information on this area, the reader may refer to the recent reviews by Baldea and Harjunoski (2014) and Dias and Ierapetritou (2016).

As pointed out by Barbosa-Póvoa (2012), the integration of design and planning decisions is fairly well established. However, the integration of planning and scheduling remains an open problem. Maravelias and Sung (2009)

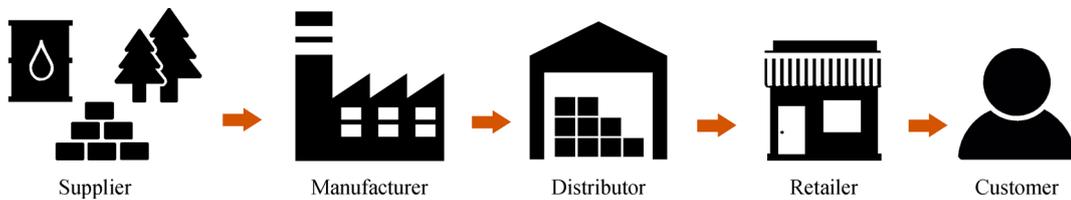


Fig. 1 Supply chain example

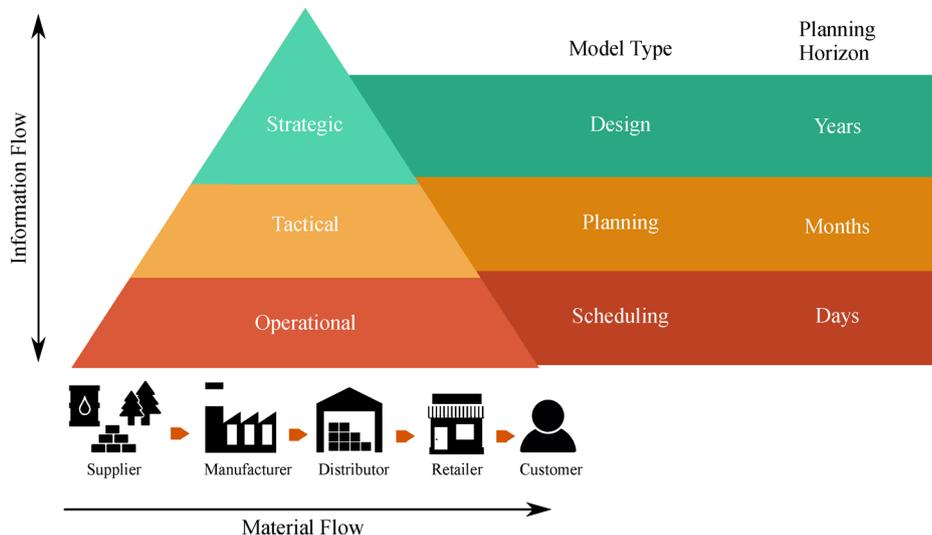


Fig. 2 Supply chain decision pyramid

have recently published a review paper on this topic. Garcia and You (2015) identify the challenges in dealing with multilevel systems, modeling, optimization, uncertainty handling, and efficient algorithms design. Although remarkable progress has been achieved in examining the elements required to handle multilevel systems, the integration of these elements remains to be addressed. Figure 3 presents a matrix of the identified models, which are classified by the decision level and stage of the supply chain.

The current practice is to execute each planning process independently. Integration allows companies to be highly agile when reacting to the dynamic conditions of the process environment, capturing additional benefits. For example, the timing of making strategic decisions is usually fixed to once a year, and the decisions are treated as static during this period. Having integrated models enables one to identify opportunities in which strategic planning must be pulled forward to adapt to changing conditions and to advance toward online optimization and supply chain control (Perea-López et al., 2001).

The decision hierarchy also impacts the organizational structure of a company. Managers make decisions that impact the actions of their subordinates. Given that the majority of the conflicts in an enterprise are related to communication, achieving a better integration of decision-making processes can improve the work environment by enhancing the coordination among different functional areas. These benefits are hard to measure but are certainly present.

This paper presents an overview of all elements involved in multilevel optimization by selecting some examples from the literature and tries to identify the open challenges in achieving overall integration. Section 2 presents a network representation for this problem. Sections 3, 4, and 5 analyze the models for design, planning, and scheduling, respectively. Section 6 explores the ways for coordinating different decision levels. Section 7 summarizes the challenges in achieving multilevel integration.

## 2 Modeling structure network

Supply chains are systems with multiple components that exchange information with one another. Thus, a network representation is ideal to represent the structure and interactions between each decision-making component. In this section, we propose a standard structure and outline the elements that need to be considered when optimizing the whole supply chain. The network representation has motivated Jalving et al. (2017), who developed a computational package called PLASMO for the Julia programming language (Bezanon et al., 2012), to represent networks of models with the JuMP mathematical programming platform (Dunning et al., 2017).

Figure 4 presents the proposed structure, where each node represents the problem that each decision maker at a defined level needs to solve. Each solid-line arc represents the influence of top-level decisions in the sub-nodes. The relationship at an arc can be defined as passing the value of a variable or a constraint that involves the variables of the origin and destination nodes. If the network is solved by simply optimizing each node from the root node to the leaves, then not even the feasible solutions can be guaranteed. The feedback (represented by the dashed-line arcs) is necessary to converge to the optimal solution. Section 6 presents additional information on the coordination among nodes.

We use the following example to explain these definitions further. In a chemical company, the global planner from a family of specialty chemicals allocates the production of each of product in his/her portfolio to each production area (Fig. 5). When doing the allocation, the global planner considers the transfer between different production areas. Afterward, the planner for each production area must decide how to split the global planner requirement among different manufacturing plants in the production area. In each plant, a scheduler will perform short-term planning to execute the manufacturing operations and to meet the required targets. The optimal solution

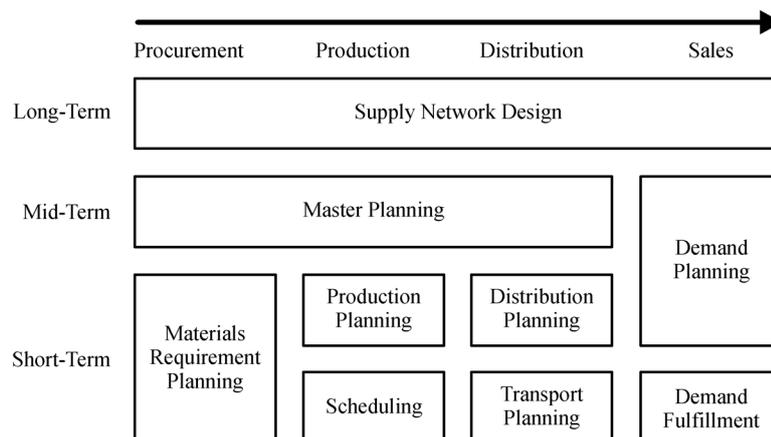


Fig. 3 Planning matrix

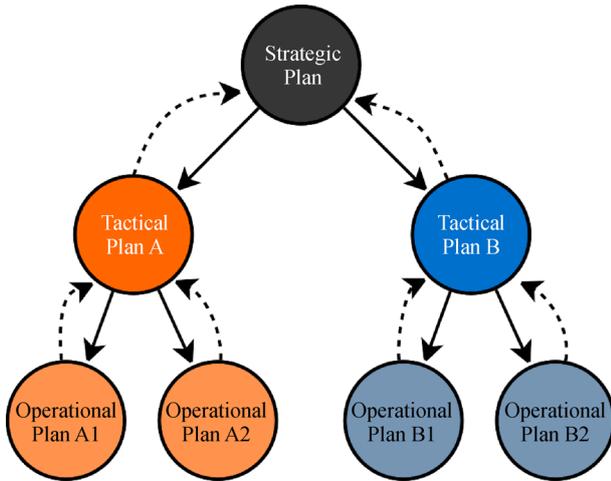


Fig. 4 Supply chain decision network structure

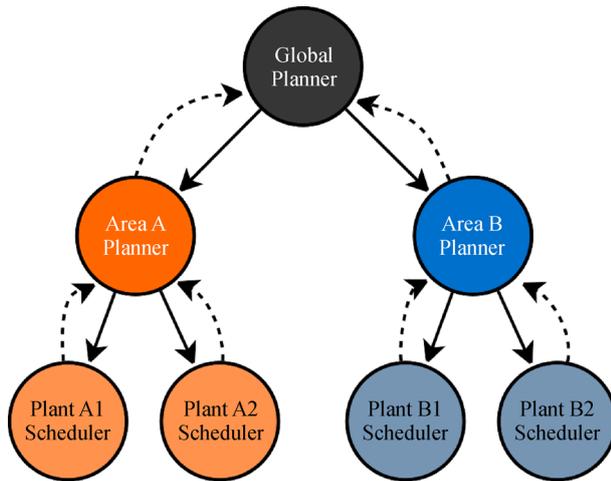


Fig. 5 Example of a supply chain decision network structure

for the network is unlikely to be obtained without feedback and iteration. We refer to the optimal solution of the network ( $x^*$ ), or the integrated solution, to the vector of variables that maximizes the benefit across the whole network:

$$x^* = \arg \max_{x \in X} \sum_{n \in N} z_n, \tag{1}$$

where  $z_n$  represents the objective function of each node  $n$ .

This definition of integrated solution differs from that in the literature, which usually considers only two decision levels and not even in their full detail. Previous studies usually include a relaxation or aggregation of the subproblem as a constraint of the top-level problem (Maravelias and Sung, 2009) to improve the quality of the solution for the top level. However, the value of the lower decision variables is not meant to be implemented (Fig. 6). Some authors have called this approach an integrated solution.

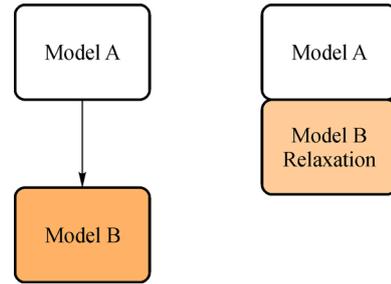


Fig. 6 Concept of integrated solution as defined in the literature

Considering the structure shown in Fig. 4, an optimal solution for the network can be obtained by answering the following questions:

- (1) What is the right model and solution method for each node?
- (2) How to coordinate the decisions that are made between nodes?

For the first question, much progress has been achieved by developing models for different applications and decision levels (Bixby et al., 2004). Solution strategies based on mixed-integer programming (Linderoth and Savelsbergh, 1999), constraint programming (Hooker, 2002), metaheuristics (Blum and Roli, 2003), and others, have also been proposed. Handling uncertainty is also important in obtaining implementable solutions (Birge and Louveaux, 2011). Sections 3, 4, and 5 analyze the details related to the first question, while Section 6 addresses the second question mainly through decomposition algorithms.

### 3 Modeling of strategic decisions

Strategic decisions define the long-term plans that affect all areas of a company. These decisions represent commitments that span for many years, such as the execution of capital projects and contracts. The impact of these decisions is so significant that failing to make good decisions can lead to the demise of the company. In order to make these critical decisions, several years of forecast data must be considered. However, given that one may consider points in time that are very distant to the present, these forecasts may become highly uncertain.

The decisions at the strategic level are generally classified as follows:

- 1) Product portfolio selection;
- 2) Contracts; and
- 3) Facility installation, expansion, and reduction.

For product portfolio selection, a company decides which of its products must be maintained in its portfolio based on the projected profitability of serving the forecasted demand. The research on these problems has focused on improving the current forecasting methods

(de Weck et al., 2003) and on handling the new products entering the pipeline (Jain and Grossmann, 1999). The demand depends on the phase of the life cycle in which a product is found (Fig. 7).

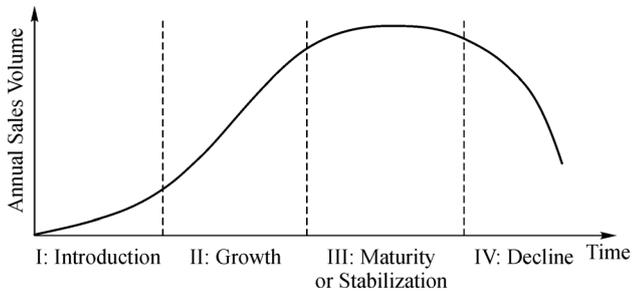


Fig. 7 Product life cycle curve

From the modeling standpoint, the optimal portfolio can be obtained by allowing the model not to meet the total demand (Eq. 2). However, those cases in which the demands of products are coupled must also be considered. For example, a company may be supplying an unprofitable product to a customer to keep serving that same customer with other highly valuable products (Rastogi et al., 2011).

$$\sum_{j \in J_k} x_{jkpt} \leq \text{Demand}_{kpt} \quad \forall k \in K, p \in P, t \in T. \quad (2)$$

In Eq. (2),  $K$  is the set of customers,  $P$  is the set of products,  $T$  is the set of time periods,  $J_k$  is the set of facilities serving customer  $k$ , and  $x_{jkpt}$  is the flow of material of product  $p$  between facility  $j$  and customer  $k$ . Uncertainty is present in the demand forecast for both new and mature products.

Contracts are also important strategic decisions that dictate the prices and restrictions on the amounts related to supply, sales, and transportation. Supply and sales contracts have been modelled in Park et al. (2006), who consider the following types of contracts:

**Fixed price contracts**, where the purchase price is fixed and is independent from the amount purchased.

**Discount after a certain amount**, where the first  $\sigma_1$  units are sold to a price  $\varphi_1$ , while the units in excess of  $\sigma_1$  are sold to a price  $\varphi_2$ .

**Bulk discount**, in which the price for the entire order decreases to  $\varphi_2$  when the amount ordered exceeds  $\sigma_1$ .

**Fixed duration contracts**, where a minimum purchase and contract duration are defined. The longer the contract, the larger the minimum purchase and the lower the price.

All contract options can be modeled using generalized disjunctive programming (Grossmann and Trespacios, 2013) and be formulated as mixed-integer linear programming (MILP) models via convex hull reformulation (Balas, 1998). The same four types and conditions are used for sales contracts. The application of these formulations has been demonstrated in Drouven and Grossmann (2016),

who optimize a strategic plan for shale gas extraction by considering different kinds of contracts. Transportation contracts have been optimized by Yano (1992). In addition to amounts and prices, these contracts consider multiple transportation modes and the provision of urgent services.

Capacity installation, expansion, and contraction have also been widely studied. Martínez-Costa et al. (2014) review the strategic models related to capacity expansion and describe the main decisions and factors involved in strategic planning. Their reviewed models involve tactical decisions, such as those related to production and demand allocation. Martínez-Costa et al. apply the same approach described in Fig. 6 and consider the location of facilities in their analysis. With the exception of temporary facilities, all problems related to facility location yield strategic models, which have been comprehensively reviewed in Melo et al. (2009). Capacity increases may take the form of simple equipment purchases or highly complex construction projects, such as installing new plants or building new construction sites. However, they do not make a particular distinction between these cases.

To better understand the modeling approach for strategic decisions, we use two examples from the literature on the process industry. The first example is the model proposed by Levis and Papageorgiou (2004) to optimize a pharmaceutical supply chain by planning clinical trials and manufacturing capacity. The planning horizon is set to 10 years divided in one-year periods. The manufacturing capacity is controlled while deciding the number and timing of installing production lines. The installation of the first production line at a site, which is called header suite by the authors, must include the installation of general services. The capacity decisions are translated into the available production time, which restricts the production and sales. The objective is to maximize the expected net present value given the uncertainty in the success of clinical trials.

The model considers the following constraints in handling the expansion decisions:

1) A production line exists if it was present in the previous period or if it was decided to be installed before considering the construction lead time.

$$y_{it} \Rightarrow y_{it-1} \vee b_{it-\lambda_i}, \quad (3)$$

where  $i$  is the production suite,  $t$  is the time period,  $\lambda_i$  is the construction time of suite  $i$ ,  $y_{it}$  indicates the existence of suite  $i$  in period  $t$ , and  $b_{it}$  indicates that the construction of suite  $i$  starts at period  $t$ .

2) A non-header suite can only be installed if a header suite was installed before.

3) A symmetry breaking constraint; suite  $i-1$  must be installed before suite  $i$ .

The second group of constraints includes production, inventory, and sales constraints. Interestingly, the sales are bounded by the demand, thereby allowing the model to

perform portfolio planning.

$$\text{sales}_{kpt} \leq \text{Demand}_{kpt} \quad \forall k, p, t. \quad (4)$$

The rest of the constraints are specific to the pharmaceutical industry. A scale-up operation is required when a product is manufactured at a plant for the first time, and then qualification runs must be performed to show that the plant is operating in compliance to the regulations. To solve the resulting model, this study proposes a hierarchical algorithm in which the strategic decisions are taken first with aggregated production and then the tactical plan is solved in a reduced space.

The second example is the problem studied by You et al. (2011). The objective of the model is to determine the expansion strategy of multiple chemical sites, together with supply chain planning. The considered planning horizon is 10 years divided in by year. Each site has a defined number of slots in which a production train can be installed. Each train corresponds to a reactor and the associated downstream facilities. The size of the production trains, including those that are installed and are available for installation, is predefined. In addition to the timing and size of expansions, the model decides to which production family must the production train be dedicated. Apart from expansion, the options for shutting down or converting a production train into another product family are considered.

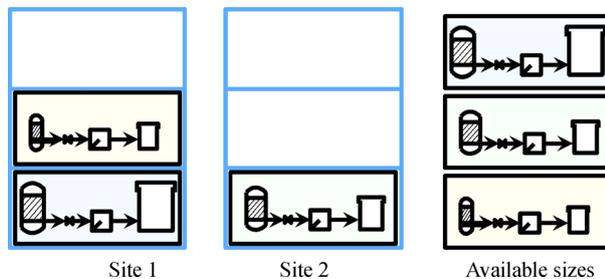


Fig. 8 Site expansion modeling

The model considers three processes related to capacity, namely, installation, shutdown, and transformation of production trains into another product family. Blocks of logical constraints are defined for each process. The model also considers tactical planning decisions at the product level. Given that the strategic decisions are made at the strategic level, appropriate conversions are included in the production constraints. Given the model complexity, the authors develop two solution strategies, namely, bilevel decomposition (Iyer and Grossmann, 1998) and Lagrange decomposition (Guignard and Kim, 1987). Similar to the first example, the sales variables are bound by demand. However, in this model, the sales must be within a certain demand range (Eq. 5).

$$\text{MinDemand}_{kpt} \leq \text{Sales}_{kpt} \leq \text{MaxDemand}_{kpt} \quad \forall k, p, t. \quad (5)$$

The following similarities and differences are observed between these examples:

1) The capacity is discrete in both examples, and such capacity can be increased or decreased by changing the number of capacity units (i.e., manufacturing suites in the first example and production trains in the second example). The constraints related to capacity include binary or integer variables and logical relations.

2) Both examples consider yearly periods and a 10-year planning horizon, which are considered appropriate for strategic decisions. However, for the tactical decisions considered, the time discretization yields aggregated plans. The time grid must be refined further to accurately optimize the tactical decisions.

3) The first example only considers expansion, while the second example considers expansion, shutdown, and transformation.

4) The effect of installing the first capacity unit is considered in the first example.

5) The demand satisfaction constraints are treated as inequalities in both examples, thereby making these cases suitable for portfolio optimization. For the second example MinDemand must be set to zero.

Given the discrete nature of strategic decisions, these problems usually yield MILP models that are solved either with standard branch-and-cut solvers or heuristics. The models in the literature show some similarities with the above examples. However, no general modeling approach has been defined yet.

Stochastic programming is preferred when handling uncertainty at the strategic level because longer planning horizons are assumed to provide more opportunities for reevaluating decisions and take recourse actions (Snyder, 2006). Flexibility analysis has been proposed in process engineering design (Swaney and Grossmann, 1985) because of its strong connections to robust optimization (Zhang et al., 2016). However, its application to supply chain design has been limited (Mansoornejad et al., 2011; Sahay and Ierapetritou, 2015; Wang et al., 2016).

## 4 Supply chain planning

The optimization of tactical decisions has been investigated in supply chain planning research. With the strategic decisions already fixed, the goal is to optimize the material flows and inventories in the supply chain network. The lot-sizing problem is used as the base model to represent this decision level (Karimi et al., 2003). The planning horizon ranges from six months to a couple of years divided into monthly periods. The flow among different echelons (i.e., suppliers, manufacturing sites, distribution centers, and customers) in the network must be examined (Fig. 9). Given that the models are multiperiod, the inventory levels are simultaneously determined.

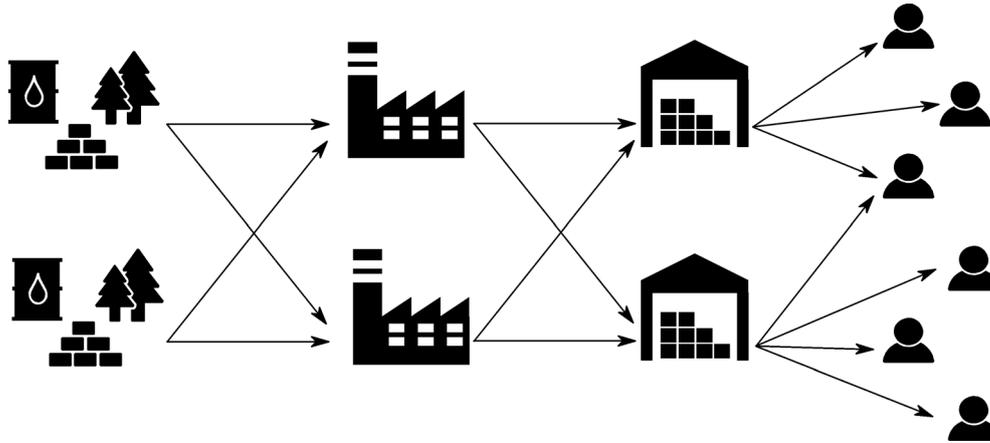


Fig. 9 Supply chain planning network

The basic constraints of the problem include inventory balance (Eq. 6), and constraint satisfaction (Eq. 7), which are defined as follows:

$$inv_{jpt} = inv_{jpt-1} + \sum_i x_{ijpt} - \sum_k x_{jkpt} \quad \forall j, p, t, \quad (6)$$

$$\sum_j x_{jkpt} = Demand_{kpt} \quad \forall k, p, t, \quad (7)$$

where  $i$  is a manufacturing site,  $j$  is a distribution center,  $k$  is a customer,  $p$  is a product, and  $t$  is a time period. The variables  $inv$  and  $x$  represent inventory and flow, respectively.

A large variety of application-specific constraints complement these models. In the simplest case, the problem yields LP models. However, MILP models are frequently found in the literature when start-up or fixed transportation costs are considered. NLP models are also defined in the presence of material blending, such as oil blending problems (Lotero et al., 2016). To handle uncertainty, the inventory optimization problem that considers safety stocks has been examined (You and Grossmann, 2011).

## 5 Process scheduling

After completing the mid-term planning, short-term scheduling must be conducted to plan the execution of manufacturing operations. The scheduling horizon spans from two days to four weeks, and the horizon is divided into days or even hours. General modeling frameworks have been defined to address this problem. The resulting models are difficult to solve, thereby motivating intensive research in this area. Harjunkoski et al. (2014) and Méndez et al. (2006) present comprehensive reviews of the models and applications of process scheduling. We focus on analyzing the general characteristics of modeling frame-

works to identify the key elements for multilevel optimization.

Time representation presents a major challenge in scheduling models (Floudas and Lin, 2004). Discrete time is often used for tracking resource constraints. Although continuous time is more accurate than discrete time, computing the former is more difficult than computing the latter.

Proposed by Kondili et al. (1993), the state–task–network (STN) formulation involves the transformation of materials (called states) by employing tasks. Figure 10 presents an example of an STN. In the diagram, each circle represents a state and each rectangle represents a task. The process stoichiometry is represented by the coefficients at each arc. Although not included in the diagram, process equipment is considered in the model.

The main decision variables include the following:

- 1)  $y_{ijt}$ : a binary variable indicating if task  $i$  is performed in equipment  $j$  at the beginning period  $t$ ;
- 2)  $b_{ijt}$ : the size of the batch processed at equipment  $j$  in period  $t$ , executing task  $i$ ;
- 3)  $s_{kt}$ : the inventory of state  $k$  in period  $t$ .

The following groups of constraints are also considered: **Logical constraints** ensure that no more than one task is assigned to an equipment at a given period and that no task is assigned while the equipment is executing a task.

**Material balances**, which are required for each state. Each circle in Fig.10 represents a storage tank.

The minimal STN model with discrete time representation is given as followings:

$$\text{Max} \sum_k \eta_k s_{kT}, \quad (8)$$

$$\text{s.t.} \sum_{i \in I_j} \sum_{\tau=t-PT_{ij}+1}^t y_{ij\tau} \leq 1 \quad \forall j, t, \quad (9)$$

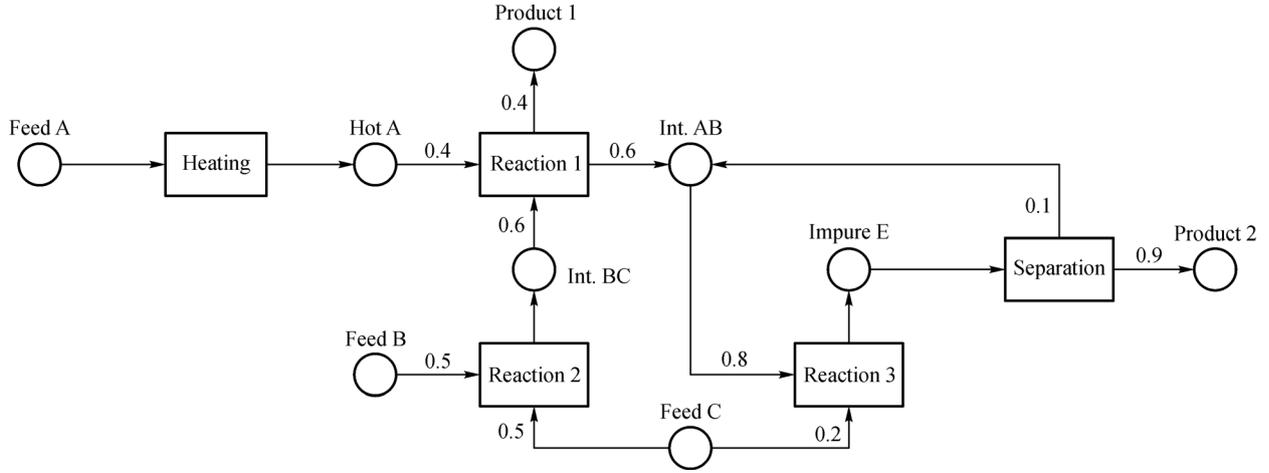


Fig. 10 State-task-network example

$$s_{kt} = s_{kt-1} + \sum_{i \in I_k^p} \rho_{ik}^p \sum_{j \in J_i} b_{ij(t-PT_{ij})} - \sum_{i \in I_k^c} \rho_{ik}^c b_{ijt} + \Pi_{kt} - D_{kt} \quad \forall k, t, \quad (10)$$

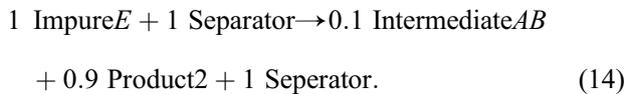
$$V_{ij}^{\min} y_{ijt} \leq b_{ijt} \leq V_{ij}^{\max} y_{ijt} \quad \forall i, j \in J_i, t, \quad (11)$$

$$C_k^{\min} \leq s_{kt} \leq C_k^{\max} \quad \forall k, t, \quad (12)$$

$$b_{ijt}, s_{kt} \geq 0, y_{ijt} \in \{1, 0\}. \quad (13)$$

The objective value is to maximize the profit involving the final inventory  $s_{kT}$  in period  $T$ . The parameter  $\eta_k$  is the inventory value.  $\Pi_{kt}$  and  $D_{kt}$  denote the planned material or product deliveries,  $PT_{ij}$  is the processing time of task  $i$  in unit  $j$ ,  $\rho_{ik}^p$  and  $\rho_{ik}^c$  denote the proportion of state  $k$  produced or consumed by task  $i$ .  $V_{ij}^{\min/\max}$  is the minimum/maximum batch size, and  $C_k^{\min/\max}$  is the minimum/maximum inventory of state  $k$ .

Pantelides (1994) has proposed the resource-task-network (RTN) formulation in which the processing equipment is explicitly considered. The RTN framework is less intuitive yet more general than STN. The equipment units and materials are not distinguished because all of them are treated as resources. The stoichiometric relation for the separation task is represented as follows:



In this way, any resource including materials, equipment, utilities, and human resources, can be seamlessly incorporated into the model, thereby requiring a single balance constraint.

**Resource balance**, a balance constraint is required for

each resource. The beginning of a task is controlled by the availability of resources performing such task, including the processing equipment.

The main decision variables include the following:

1)  $y_{it}$ : a binary variable indicating if task  $i$  starts at the beginning of period  $t$ ;

2)  $b_{it}$ : the size of the batch processed in period  $t$ , executing task  $i$ ;

3)  $r_{kt}$ : the inventory of resource  $k$  in period  $t$ .

The simplest discrete time RTN model is given as follows:

$$\text{Max} \sum_k \eta_k r_{kT} \quad (15)$$

$$\text{s.t. } r_{kt} = r_{kt-1} + \sum_{i \in I_r} \sum_{\tau=0}^{PT_i} (\mu_{ir\tau} y_{i(t-\tau)} + \nu_{ir\tau} b_{i(t-\tau)}) + \Pi_{rt} \quad \forall k, t, \quad (16)$$

$$V_{ik}^{\min} y_{it} \leq b_{it} \leq V_{ik}^{\max} y_{it} \quad \forall i, r \in R_i, t, \quad (17)$$

$$b_{it}, r_{kt} \geq 0, y_{it} \in \{1, 0\}, \quad (18)$$

where  $\mu_{ikt}$  and  $\nu_{ikt}$  indicate the fixed and variable proportion of production (positive value) or consumption (negative value) of resource  $k$  for task  $i$  at interval  $\tau$  relative to the start of processing task  $i$  (Méndez et al., 2006).

Zyngier and Kelly (2012) recently proposed a new representation for scheduling problems called unit-operation-port-state-superstructure (UOPSS), which begins from the process flow diagram, a physical equipment perspective, and extends it to include logical units (operations). UOPSS includes the following types of units:

- 1) process,
- 2) pool,
- 3) pipeline,

- 4) pileline (stacks); and,
- 5) parcel.

Each unit has a defined set of constraints (Zyngier and Kelly, 2009). The UOPSS models include the following groups of constraints:

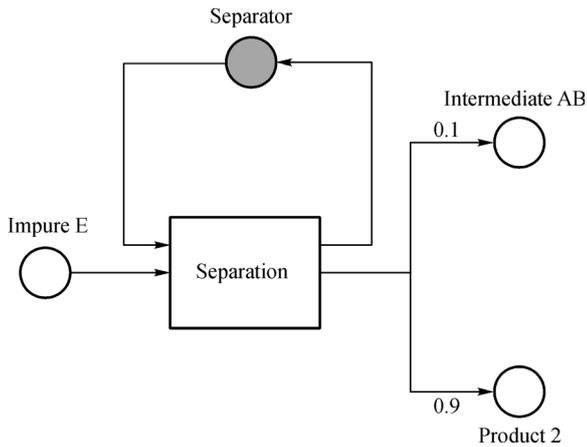
**Logical Balances**, where logical constraints involve binary variables to handle start-up, shutdown, and other logical relationships required by the model;

**Quantity Balances**, including the material balances of different states and resources present in the model;

**Logistic Balances**, including the constraints that relate quantity variables to logical variables.

For example, consider that the separator from Fig. 11 can perform another operation to split an input F into product 2 and another species G. Figure 12 shows the UOPSS representation for this case.

The triangles, rectangles, and circles in Fig. 12 represent storage tanks, operations, and ports, respectively. The



**Fig. 11** Resource-task-network representation of the separation task from Fig.10

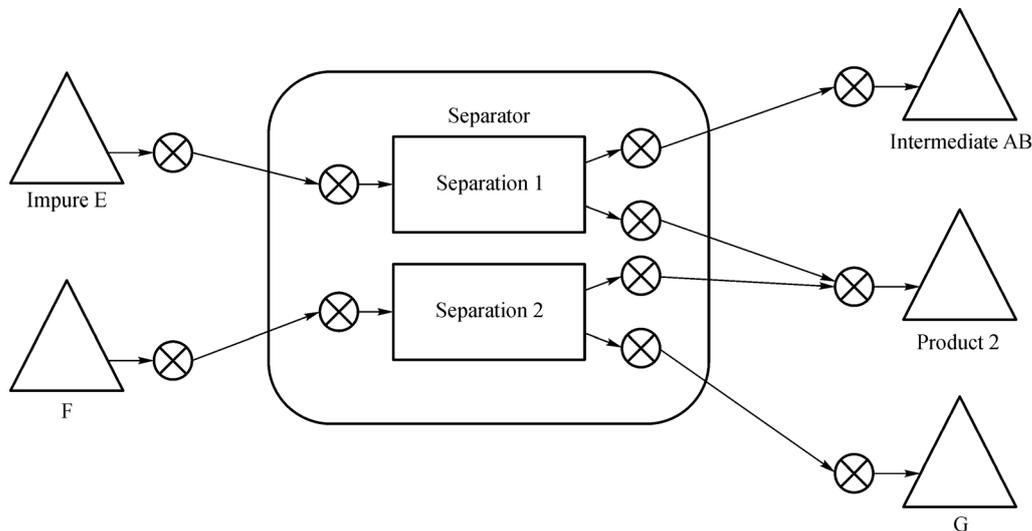
diagram displays an explicit physical connectivity between the storage tanks and the separator as well as the process perspective by including logical units (separation modes in the example). The storage tanks are pool units, whereas the separator is a process unit. Both STN and RTN have been extensively used in many applications. Given that UOPSS has been proposed much later than the two other representations, its application has not been as extensive. The similarities and differences among these three representations remain unknown.

Industrial size scheduling problems are very difficult to solve. Given the combinatorial nature of scheduling models, several solution methods have been proposed. Besides using MILP solvers, heuristics, and constraint programming methods have also been employed. Jain and Grossmann (2001) propose a hybrid method for combining MILP and constraint programming, while Maravelias and Grossmann (2004) apply this algorithm to batch scheduling.

Given that the duration and yield of operations are inherently uncertain, scheduling under uncertainty has attracted much research interest. Aytug et al. (2005) and Li and Ierapetritou (2008) have published comprehensive reviews on this topic, while Lappas and Gounaris (2016) recently propose an adaptive robust optimization framework for short-term scheduling.

## 6 Model network coordination

Several models have been recently proposed for each decision level. However, the integration of multiple decision levels has not been completely solved. In this section, we present an overview of different approaches for coordinating the solutions obtained for each node of the decision network. Optimization models are formulated to



**Fig. 12** UOPSS representation example

resolve the trade-offs between opposing decision objectives. For example, in a facility location problem, a trade-off exists between the fixed costs arising from opening facilities and the distribution costs. Opening many facilities in order to be as close as possible to the customers can increase the fixed costs for keeping more facilities open and lower the distribution costs. Therefore, a conflict arises between the objectives of different decision levels. To resolve such conflict, an optimal solution that maximizes the benefit of the sum of all objective functions must be proposed. This solution can be devised in several ways, such as using large-scale models, multiobjective optimization (MO), game theory, and decomposition.

### 6.1 Large-scale models

The first approach is to combine the models from each node into a single large-scale model. If this model can be solved, then this model yields the optimal solution for the system in a single step. However, tractability presents an evident challenge. A larger model is more difficult to solve, especially considering that the solution time scales exponentially with model size. Another downside of this approach is that the effective exploitation of parallel computing depends on the solver capabilities.

This approach is the same as if a manager is required to develop a production plan for the next year, then he/she will ask his/her three direct reports to attend a large meeting with all stakeholders involved in making the decision. Finding a solution that satisfies all meeting attendees within the duration of the meeting can be tremendously challenging. In many cases, no agreement is reached within the available time. The more participants in the meeting, the more difficult it is to reach an agreement.

### 6.2 Multiobjective optimization

MO is another approach for finding the optimal solution for the decision network. The idea behind this approach is to acknowledge the existence of two or more objectives and to determine the trade-off between them. This approach is especially useful for situations in which the objectives cannot be added because they have different units, such as when sustainability objectives are considered (Pinto-Varela et al., 2011; Guillén-Gosálbez and Grossmann, 2009). Širovnik et al. (2016) recently propose a rigorous way of combining such objectives with the concept of sustainability net present value.

The output of MO represents the variation of one objective with respect to the other. This output defines a family of Pareto optimal solutions (Fig. 13) and provides the decision maker with enough flexibility to select the operation point in the Pareto front using any desired criterion depending on the valuation given to the objectives considered.

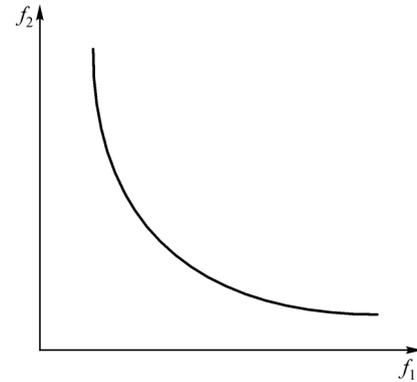


Fig. 13 Pareto front output from solving a bi-objective problem

### 6.3 Decomposition

Large-scale problems have been effectively solved by employing decomposition, in which the problem is partitioned into two or more parts that are solved iteratively and exchange information with one another. Given that the time for solving a problem increases exponentially along with the problem size, the time for conducting one iteration is far less than the time for solving the full-space problem. The success of these algorithms depends on the fact that many problems can be solved in less than a couple hundred iterations in a fraction of the time required to solve the full-space problem.

In Benders decomposition (Benders, 1962), the problem is solved iteratively by exchanging information among the decision makers. The problem is partitioned between a master problem and one or more subproblems. The master problem proposes a value for its variables, and the subproblems devise their best solution based on the proposed values that they receive. Therefore, the subproblem is a function of the variables of the master problem. By considering only part of the full problem, the master problem yields a lower bound of the optimum (in the case of minimization). The subproblems yield a full feasible solution, an upper bound of the optimum. The iterations continue until both bounds match the desired tolerance.

Going back to the analogy of the manager needing to devise a production plan for the next year, Benders decomposition corresponds to the process in which the manager and his/her reports work separately in making the decisions within their scope. They iteratively exchange the results of their decision-making process with one another until they reach an agreement. This process demonstrates how the decisions are made in practice. However, only few iterations are performed in actual scenarios, thereby offering many opportunities for improving the current decision-making processes. Interestingly, the Benders decomposition can also represent bargaining processes and establish strong connections with game theoretical approaches.

Mathematically, the objective value of the subproblems is a function of a decision that is made by the master problem. Those problems with a block angular structure as shown in Fig. 14 are amenable to the Benders decomposition because selecting the linking variables as part of the master problem will decompose the problem into several subproblems that can be solved in parallel. This case is observed in two-stage stochastic programming.

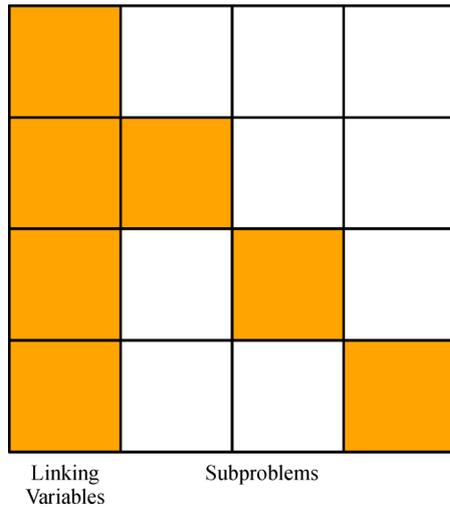


Fig. 14 Block angular structure with linking variables

To clarify these concepts, consider the following example:

$$\begin{aligned}
 P : \text{Min} \quad & c^T x + d^T y \\
 \text{s.t.} \quad & Ax \leq b \\
 & Ey \leq f \\
 & Gx + Hy \leq q \\
 & x \in X, y \in Y.
 \end{aligned} \tag{19}$$

The problem  $P$  can be partitioned into a master problem and a subproblem. For example,  $x$  may represent the vector of strategic variables, while  $y$  may represent the vector of tactical variables. The strategic problem (or master problem) is then defined by Eq. 20, while the subproblem is defined by Eq. 21,

$$BM : \text{Min} \quad c^T x + \theta, \tag{20a}$$

$$\text{s.t.} \quad Ax \leq b, \tag{20b}$$

$$\theta \geq \theta^k - \lambda^k (x^k - x) \quad k = 1, \dots, K, \tag{20c}$$

$$g(x, x^l) \leq 0 \quad l = 1, \dots, L, \tag{20d}$$

$$\theta \geq \theta^{LB}, \tag{20e}$$

$$x \in X, \theta \in \mathbb{R}, \tag{20f}$$

$$SP(x) : \text{Min} \quad d^T y, \tag{21a}$$

$$\text{s.t.} \quad Ey \leq f, \tag{21b}$$

$$G\hat{x} + Hy \leq q, \tag{21c}$$

$$x - \hat{x} = 0, \tag{21d}$$

$$y \in Y. \tag{21e}$$

The Benders master problem optimizes the strategic decisions and acknowledges that part of the problem is unknown to itself (the tactical problem) by including the variable  $\theta$  in the objective function. The feasible space for this variable is iteratively approximated by a family of cuts (Eq. 20c). The variable  $\theta$  is bounded by  $\theta^{LB}$ , otherwise the problem  $P$  will be unbounded. The objective value of subproblem SP is a function of vector  $x$ . In other words, the tactical decision depends on the strategic decision. At iteration  $k$ , a value for  $x$  is given and the optimal solution for SP is determined. From the subproblem, the objective function and  $\lambda^k$ , which are the dual variables of Eq. 21d, are used to construct the cut from Eq. 20c. This process corresponds to the coordination–feedback process described in Mesarovic et al. (1970).

When the subproblem is a linear programming problem (LP), the duals  $\lambda^k$  are well defined and the cut from Eq.20c can be easily obtained. However, when the subproblem is an MILP, the duals are not defined and additional efforts are required to obtain tight cuts. Mathematically speaking, finding the optimum decision network is equivalent to solving a multistage stochastic programming problem with mixed-integer recourse (MSMIP). When Fig. 4 is rotated by 90°, the usual representation for a multistage scenario tree is obtained. Therefore, all the methods developed for solving MSMIPs are applicable to multilevel systems.

Feasibility is the simplest feedback that can be provided by a subproblem. The decision proposed by the master problem can be either feasible or infeasible. When the subproblem is infeasible, a cut to exclude such solutions must be incorporated in the master problem (Eq. 20d). When the subproblem is an LP, the cut is given by Eq. 22, where  $\nu^l$  is the Farkas proof of infeasibility, which is an unbounded extreme ray of the dual problem.

$$0 \leq \nu^l (x^l - x). \tag{22}$$

When the subproblem is a mixed-integer problem, the duals are not defined, and other cuts need to be defined. Balas and Jeroslow (1972) propose “no-good” cuts for the case of binary master variables (Eq. 23)

$$\sum_{i: x_i^l=0} x_i + \sum_{i: x_i^l=1} (1 - x_i) \geq 1. \tag{23}$$

When the subproblem is feasible, the feedback is given

in terms of the objective value and the dual information. When the subproblem is an LP a standard Benders cut is generated (Eq. 24).

$$\theta \geq \theta^k - \lambda^k (x^k - x). \quad (24)$$

When the subproblem is a mixed-integer problem, the dual information becomes unavailable and additional considerations are required to generate cuts. The first option only uses the objective value. Laporte and Louveaux (1993) propose the cut from Eq. 25 for the case of binary master variables.  $\theta^{\text{LB}}$  represents a lower bound of the objective value of the subproblem.

$$\theta \geq \theta^k - (\theta^k - \theta^{\text{LB}}) \left( \sum_{i: x_i^k=0} x_i + \sum_{i: x_i^k=1} (1 - x_i) \right). \quad (25)$$

The cut is tight at the optimal solution, but is very weak in general. This cut pushes the master problem to agree with the subproblem in terms of the value of  $\theta^k$  unless it changes the components of the master variables vector  $x$ . The number of components that need to be changed to “override” the effect of the cut, depends on the quality of the bound  $\theta^{\text{LB}}$ . In the worst case, when  $\theta^{\text{LB}} = 0$ , the cut becomes useless by changing only one component.

The second option is to solve the convex relaxation of the subproblem to approximate the objective value and obtain dual information. Zou et al. (2017) propose three additional cut options for the case of binary master variables.

**Benders Cut.** The cut from Eq. 24 can be generated by solving the LP relaxation of the subproblem. The cuts are valid and finite. However, using these cut does not guarantee convergence to the optimal solution.

$$\theta \geq \theta^{\text{LP}} - \lambda^{\text{LP}} (x^k - x). \quad (26)$$

**Lagrange Cut.** The subproblem can be relaxed and a Lagrange relaxation can be defined by dualizing Eq. 21d. Given that the Lagrange relaxation is at least as tight as the LP relaxation, the obtained cut dominates the Benders cut. However, obtaining this cut is more expensive than obtaining the Benders cut.

$$\theta \geq \theta^{\text{LR}} - \lambda^{\text{LR}} (x^k - x). \quad (27)$$

**Strengthened Benders Cut.** An intermediate option between Benders cut and Lagrange cut is using the dual values from the LP relaxation to initialize the Lagrange relaxation. The objective value of the first iteration can be used to define a cut. This family of cuts is valid and finite but does not necessarily dominate the Benders cut.

$$\theta \geq \theta^{\text{LR}_1} - \lambda^{\text{LP}} (x^k - x). \quad (28)$$

In the above equations, a superindex LP indicates the LP relaxation, LR indicates the Lagrange relaxation, and LR<sub>1</sub> indicates the first iteration of the Lagrange relaxation.

Gade et al. (2014) propose adding Gomory cuts when the integer variables of the subproblem take a fractional value in the LP relaxation. By successively adding cuts, the objective value of the relaxation is strengthened, and dual information becomes available. Sherali and Fraticelli (2002) generate cuts by applying the relaxation-linearization technique, which involves lifting the space by considering one binary variable at a time. However, such procedure only limits the application of this technique to small problems.

All the cut options proposed for mixed-integer subproblems are not tight enough, expensive to obtain, and limited to binary master variables, thereby posing a challenge in multilevel optimization.

However, this problem can be compensated by exploiting parallel computing. Each problem that can be solved with Benders decomposition can also be solved by Lagrange decomposition (Guignard and Kim, 1987), by disaggregating the master variables, and by adding non-anticipativity constraints. If the problem  $P$  (Eq. 19) decomposes into  $|I|$  subproblems, then the  $y$  variable can be written as  $y_i : i \in I$ , where  $y_i$  is the local variable corresponding to subproblem  $i$ . In principle, vector  $x$  does not have a component for each subproblem but the variables can be duplicated by adding non-anticipativity constraints.

$$x_i : i \in I, \quad (29)$$

$$x_i - x_{i+1} = 0 \quad \forall i \in I. \quad (30)$$

The constraints from Eq. 30 can be dualized to apply Lagrange decomposition. Given that a multilevel optimization problem can be solved with both Benders and Lagrange decomposition, both algorithms can be run in parallel and exchange information with each other, which is the very idea of cross decomposition (Van Roy, 1983; Mitra et al., 2014).

#### 6.4 Game theory approaches

As briefly discussed in the previous section, finding the optimal solution of a decision network is equivalent to finding the solution of a bargaining process. These approaches have only been used to identify optimal decisions in the presence of external decision makers, such as the coordination between multiple echelons (Zamarripa et al., 2013) and the coordination between enterprises and customers (Garcia-Herreros et al., 2016). Florensa et al. (2017) showed that these approaches can also be applied to multilevel systems. The modeling of these problems creates bilevel programming models (Vicente and Calamai, 1994), in which the subproblems are embedded as constraints in the master problem. Equation 31 presents an example of a bilevel programming problem.

$$\begin{aligned}
 BPP : & \text{Min } c^T x \\
 & \text{s.t. } Ax \leq b \\
 & \text{Min } d^T y \\
 & Ey \leq f \\
 & Gx + Hy \leq q \\
 & x \in X, y \in Y.
 \end{aligned} \tag{31}$$

The problem can be solved by replacing the inner problem by its Karush–Kuhn–Tucker conditions and reformulating the resulting problem (Colson et al., 2005). The application of bilevel programming to the integration of supply chains remains uninvestigated.

## 7 Challenges

Multilevel supply chain optimization is required to design enterprise-wide decision systems. The decision network representation is much more than a useful representation for modeling and solving the problem but also provides a natural representation of how companies are organized to make decisions. This argument is reflected in the similarity between the organizational structures and the decision structures discussed in Section 2. Efficiently modeling and solving multilevel systems will produce highly responsive and competitive supply chains that make excellent decisions. The following challenges in this area also need to be addressed.

The first challenge in multilevel optimization is the unavailability of a modeling platform that allows the model to seamlessly represent the nodes of the decision structure as building blocks and their connections. The development of PLASMO (Jalving et al., 2017) is a step in the right direction, but this software is yet to be released.

The second challenge is the lack of standardized models for strategic and tactical decisions. Multipurpose general frameworks are available for process scheduling. However, how to achieve the same thing in higher levels of the decision-making pyramid remains unknown. These generalizations must also consider an appropriate aggregation of lower levels to ensure that feasibility is maintained and to accelerate the solution process. Generalization can be especially challenging at the tactical level due to the variety of conditions found in each application. However, the availability of commercial software for this purpose (Funaki, 2009) indicates that such generalization is possible.

The third challenge lies in coordination. The available algorithms are applied to the specific mathematical structures of each node. The feedback process (cut generation) is the most difficult part of the procedure that necessitates the use of decomposition algorithms that can deal with any type of node, such as nonconvex mixed-

integer nonlinear problems. The recent advancements in this area have been driven by the research in stochastic programming with mixed-integer recourse.

The fourth challenge lies in the uncertainty that is present in every decision-making problem. Such uncertainty can be addressed in several ways, such as stochastic programming, robust optimization, and flexibility analysis. The modular modeling in multilevel optimization allows for these models to be combined by using the most appropriate approach for each level. However, combining these models has not been attempted yet in the literature. Stochastic programming integrates well with the decision structure while maintaining the same mathematical structure and adding extra nodes to the decision network.

In sum, supply chain multilevel optimization presents an important frontier in decision making that is expected to make manufacturing processes smarter. Significant progress has been made in examining several components of the problem, but the integration of components and the solution to other challenges warrant further research.

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