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RESEARCH ARTICLE

Uncertain and multi-objective programming models for crop planting structure optimization

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Abstract Crop planting structure optimization is a significant way to increase agricultural economic benefits and improve agricultural water management. The complexities of fluctuating stream conditions, varying economic profits, and uncertainties and errors in estimated modeling parameters, as well as the complexities among economic, social, natural resources and environmental aspects, have led to the necessity of developing optimization models for crop planting structure which consider uncertainty and multi-objectives elements. In this study, three single-objective programming models under uncertainty for crop planting structure optimization were developed, including an interval linear programming model, an inexact fuzzy chance-constrained programming (IFCCP) model and an inexact fuzzy linear programming (IFLP) model. Each of the three models takes grayness into account. Moreover, the IFCCP model considers fuzzy uncertainty of parameters/variables and stochastic characteristics of constraints, while the IFLP model takes into account the fuzzy uncertainty of both constraints and objective functions. To satisfy the sustainable development of crop planting structure planning, a fuzzy-optimization-theory-based fuzzy linear multi-objective programming model was developed, which is capable of reflecting both uncertainties and multi-objective. In addition, a multi-objective fractional programming model for crop structure optimization was also developed to quantitatively express the multi-objective in one optimization model with the numerator representing maximum economic benefits and the denominator representing minimum crop planting area allocation. These models better reflect actual situations, considering the uncertainties and multi-objectives of crop planting structure optimization systems. The five models developed were then applied to a real case study in Minqin County, north-west China. The advantages, the applicable conditions and the solution methods of each model are expounded. Detailed analysis of results of each model and their comparisons demonstrate the feasibility and applicability of the models developed, therefore decision makers can choose the appropriate model when making decisions.

Keywords crop planting structure, optimization model, uncertainty, multi-objective

1 Introduction

Crop planting structure optimization is important for both irrigation water management and agricultural management[1], and is increasingly significant in agricultural water management. Crop planting structure optimization allocates the optimum planting proportion to each crop to achieve goals of increasing agricultural economic benefit and decreasing irrigation water use[2]. Crop planting structure optimization can promote irrigation water resources management and water using efficiency in order to solve the problems of water shortage and make the development of agriculture sustainable. Accordingly, it is of significance to optimize crop planting structure especially for the arid and semi-arid areas facing serious water shortage problems.

Previously, a number of optimization methods have been developed for crop planting structure planning, ranging from single-objective optimization to multiple objectives optimization, because crop structure planning involves multiple conflicting objectives such as economic, social and environmental benefits[3–8]. Linear programming (LP) has been a widely used optimization method, but the parameters, constraints and objectives of most LP models for crop planting structure are deterministic. Actually, crop planting structure planning systems are complex and there are many uncertainties in some parameters and their

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interrelationships, constraints and objectives, e.g., the spatial and temporal variations of stream conditions and irrigation quota, the fluctuations of system benefit coefficients, the grayness of system objective and constrains, and the errors in estimated modeling parameters[9]. Fuzzy mathematics programming (FMP) has been proved to be an effective and efficient tool to solve the problems where some parameters, constraints or objectives involved cannot be defined precisely[10–13]. However, crop planting structure optimization involves more than one kind of uncertainty factor, since it involves a complex system with the interaction of multiple uncertainties, e.g., fuzziness and grayness, fuzziness and randomness. Unfortunately, this has received little attention in the literature. Although there have been a number of studies about the multi-objective optimization for crop planting structure to maintain sustainable development to meet specific objectives, there is a degree of subjectivity in these studies in terms of the solution process. Therefore, it is important for decision makers to consider how to quantitatively express multiple objectives with less subjectivity about uncertainty, and how to fully consider gradual changes in selection and recognition of weights of indexes, especially when combining with uncertainty models.

This paper reports the development of five models for crop planting structure optimization to express multiple objectives and uncertainties from different perspectives, including three single-objective and two multi-objective programming models. Model 2, an inexact fuzzy chance-constrained programming (IFCCP) model, and Model 3, an inexact fuzzy linear programming (IFLP) model; both consider fuzziness in their optimization, and are based on the framework of Model 1, an interval linear programming (ILP) model, and fuzzy mathematics programming (FMP). The difference between Models 2 and 3 is in their fuzziness, i.e., the fuzziness in Model 2 is in parameters, while the fuzziness in Model 3 is in constraints and objective functions. Model 4, a fuzzy-optimization-theory-based fuzzy linear multi-objective programming (FOTB-FLMP) model considers the multiple objectives of maximizing economic and social benefits based on the fuzzy optimization theory (FOT), while Model 5, a multi-objective fractional programming (MFP) model, considers the multiple objectives of maximizing economic benefits and minimizing crop planting area. Additionally, Model 4 takes into account the fuzziness of constraints and objective functions. The five models developed were applied in crop planting structure optimization in Minqin County, north-west China, to demonstrate their feasibility and applicability. Detailed analysis and the comparison of the results of the five models are given. Decision makers can choose the appropriate models based on actual situations, which will help to optimize crop planting structure more effectively in the wake of uncertainty.

2 Model formulation

2.1 Preliminaries

2.1.1 Interval linear programming

Interval linear programming (ILP) was first developed by Huang et al.[14] to deal with the uncertainties caused by incomplete information in constraints or objective functions, without knowing the distribution information of parameters. The standard ILP model can be described as follows:

$$\begin{cases} \min\ f^{\pm} = C^{\pm}X^{\pm} \\ A^{\pm}X^{\pm} \leqslant B^{\pm} \\ X^{\pm} \geqslant 0 \end{cases} \tag{1}$$

where $A^{\pm} \in \{R^{\pm}\}^{m\times n}$, $B^{\pm} \in \{R^{\pm}\}^{m\times 1}$, $C^{\pm} \in \{R^{\pm}\}^{1\times n}$, $X^{\pm} \in \{R^{\pm}\}^{n\times 1}$; $\{R^{\pm}\}$ denotes interval number sets; $a_{ij}^{\pm} \in A^{\pm}$, $x_j^{\pm} \in X^{\pm}$, $c_j^{\pm} \in C^{\pm}$; $f^{\pm}$ denotes objective function; assuming the numbers of positive number and negative number in $c_j^{\pm}(j = 1,2,\ldots,n)$ are k_1 and k_2, respectively, so $c_j^{\pm}<0$ $(j = k_1+1,k_1+2,\ldots,n)$ and $k_1+k_2 = n$. Thus, the source model can be converted to two submodels:

Lower bound submodel

$$\begin{cases} \min f^{-} = \sum_{j=i}^{k_1} c_j^{-}x_j^{-} + \sum_{j=k_1+1}^{n} c_j^{-}x_j^{+} \\ \sum_{j=1}^{k_1} |a_{ij}|^{\pm} sign(a_{ij}^{\pm})x_j^{-} + \sum_{j=k_1+1}^{n} |a_{ij}|^{-} sign(a_{ij}^{-})x_j^{+} \leqslant b_i^{+},\ \forall i \\ x_j^{\pm} \geqslant 0,\ j = 1,2,\ldots,n \end{cases} \tag{2}$$

By solving the above model, f_{opt}^{-}, $x_{jopt}^{-}(j = 1,2,\ldots,k_1)$ and $x_{jopt}^{+}(j = k_1+1,k_1+2,\ldots,n)$ can be obtained.

Upper bound submodel

$$\begin{cases} \min f^{+} = \sum_{j=i}^{k_1} c_j^{+}x_j^{+} + \sum_{j=k_1+1}^{n} c_j^{+}x_j^{-} \\ \sum_{j=1}^{k_1} |a_{ij}|^{-} sign(a_{ij}^{-})x_j^{+} + \sum_{j=k_1+1}^{n} |a_{ij}|^{+} sign(a_{ij}^{+})x_j^{-} \leqslant b_i^{-},\ \forall i \\ x_j^{+} \leqslant x_{jopt}^{-},\ \forall i = 1,2,\ldots,k_1 \\ x_j^{-} \leqslant x_{jopt}^{+},\ j = k_1+1,k_1+2,\ldots,n \\ x_j^{-} \geqslant 0,\ j = 1,2,\ldots,n \end{cases} \tag{3}$$

Similarly, f_{opt}^{+}, $x_{jopt}^{+}(j = 1,2,\ldots,k_1)$ and $x_{jopt}^{-}(j = k_1+1,k_1+2,\ldots,n)$ can be obtained. The final results are $x_{jopt}^{\pm} = [x_{jopt}^{-}, x_{jopt}^{+}]$, $\forall j$ and $f_{jopt}^{\pm} = [f_{jopt}^{-}, f_{jopt}^{+}]$.

2.1.2 Fuzzy linear programming

Fuzzy linear programming (FLP) blurs the constraints and objective function of LP to deal with fuzzy uncertainties[15]. FLP models are uncertain and it is necessary to transform the FLP model into a deterministic one during the solutions. The FLP can be converted to ordinary LP by introducing a membership function. Generally, the FLP model[16] can be described as:

$$\begin{cases} \max\ F = \sum_{j}^{J} c_j a_j \tilde{\geqslant} Z_0 \\ \sum_{j=1}^{J} w_{ij} a_j \tilde{\leqslant} b_i \quad \forall i \\ a_j \geqslant 0 \\ i = 1,2,\ldots,m,\ j = 1,2,\ldots,n \end{cases} \tag{4}$$

where F is the objective function; Z_0 is the optimal solution of the corresponding ordinary LP, i.e., Eq. 4 without considering the fuzzy relation inequalities; $a = \{a_1,a_2,\ldots,a_n |\ a_1,a_2,\ldots,a_n \in R^n,\ a_1,a_2,\ldots,a_n \geqslant 0\}$ are decision variables; $c_j \in \{R\}^{n\times 1}$, $w_{ij} \in \{R\}^{m\times n}$, and $b_i \in \{R\}^{m\times 1}$ are parameters; $\tilde{D}_i$ and $\tilde{F}$ are the fuzzy sets of a corresponding to constraint i and objective function respectively; d_i and d_0 are the scaling values of constraint i and objective function, with d_0 equals the difference between the optimal value of ordinary LP under the conditions of $\sum_{j=1}^{J} w_{ij} a_j \tilde{\leqslant} b_i$ and Z_0; the membership function expression of constraint i and the corresponding diagram (Fig. 1) can be expressed as:

$$\mu_{\tilde{D}_i}(a_1,a_2,\ldots,a_n) = \begin{cases} 1 & \sum_{j}^{n} w_{ij} a_j \leqslant b_i \\ 1 - \left(\sum_{j}^{n} w_{ij} a_j - b_j\right)/d_j & b_i < \sum_{j}^{n} w_{ij} a_j \leqslant b_i + d_i \\ 0 & \sum_{j}^{n} w_{ij} a_j > b_i + d_i \end{cases} \tag{5}$$

$$\mu_{\tilde{F}}(a_1,a_2,\ldots,a_n) = \begin{cases} 1 & \sum_{j}^{n} c_j a_j > Z_0 \\ \left(\sum_{j=1}^{n} c_j a_j - Z_0\right)/d_0 & Z_0 - d_0 < \sum_{j}^{n} c_j a_j \leqslant Z_0 \\ 0 & \sum_{j}^{n} c_j a_j \leqslant Z_0 - d_0 \end{cases} \tag{6}$$

$\tilde{S}$ denotes fuzzy solution sets and $\tilde{S} = \tilde{D}_1 \cap \tilde{D}_2 \cap \ldots \tilde{D}_m \cap \tilde{F}$; and according to the maximum membership grade (λ) principle, the membership function of $a_1,a_2,\ldots,a_n$ is:

$$\begin{aligned} &\mu_{\tilde{S}}(a_1,a_2,\ldots,a_n) \\ &= \bigvee_{a_1,a_2,\ldots,a_n \in A} \left\{ \wedge \mu_{\tilde{D}_i}(a_1,a_2,\ldots,a_n) \wedge \mu_{\tilde{F}}(a_1,a_2,\ldots,a_n) \right\} \\ &= \bigvee_{a_1,a_2,\ldots,a_n \in A} \left\{ \lambda | \mu_{\tilde{D}_i}(a_1,a_2,\ldots,a_n) \geqslant \lambda, \mu_{\tilde{F}}(a_1,a_2,\ldots,a_n) \right) \geqslant \lambda, \\ &\quad i = 1,2,\ldots,m,\ \lambda \geqslant 0 \} \end{aligned} \tag{7}$$

Hence, the FLP model can be converted to ordinary LP as:

$$\begin{cases} \max\ \lambda \\ \mu_{\tilde{D}_i}(a_1,a_2,\ldots,a_n) \geqslant \lambda \\ \mu_{\tilde{F}}(a_1,a_2,\ldots,a_n) \geqslant \lambda \\ a_j \geqslant 0 \\ \lambda \geqslant 0 \\ j = 1,2,\ldots,n \end{cases} \tag{8}$$

2.1.3 Fuzzy number

If $\tilde{A}$ is the fuzzy subset of E (real set), the membership function of E can be written as $\mu_{\tilde{A}} : E \rightarrow [0,1]$ while meeting: (1) $r \in E$ and $\mu_{\tilde{A}}(r) = 1$; and (2) for any $\alpha \in [0,1]$, $\tilde{A}_\alpha = \left\{ r \in E | \mu_{\tilde{A}}(r) \geqslant \alpha \right\}$ is a closed set[17].

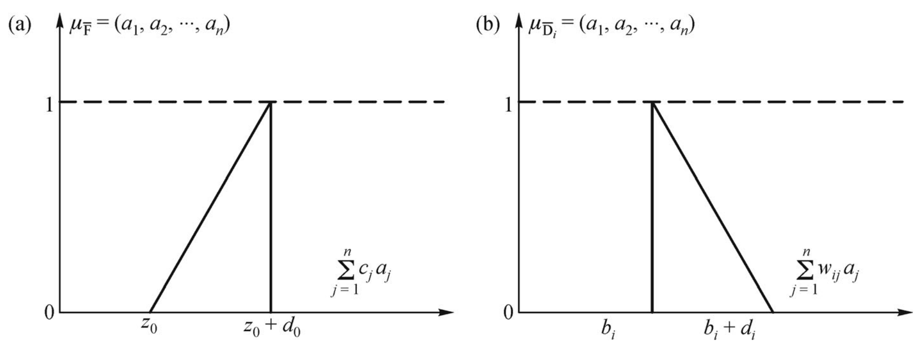

Fig. 1 Membership function of constraints (a) and objective function (b)

Taking trapezoidal fuzzy numbers as examples which are commonly used because of the handy calculation, then the two boundaries of the fuzzy number $\tilde{A} = (A_{1\min}, A_1, A_2, A_{2\max})$ can be described as $A_{\alpha\min} = (1-\alpha)A_{1\min} + \alpha A_1$ and $A_{\alpha\max} = (1-\alpha)A_{2\max} + \alpha A_2$, and the corresponding membership functionsare shown in Fig. 2. The triangle fuzzy numbers are special cases of trapezoidal fuzzy numbers. α is the level set that describes the fuzzy degree of membership level. Different α levels can quantitatively expound different levels of the possibility of events under uncertainty.

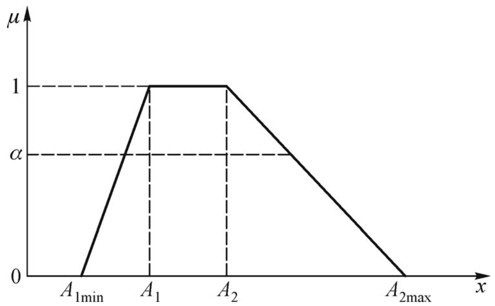

Fig. 2 Trapezoidal fuzzy membership function

2.1.4 Fuzzy chance-constrained programming

When uncertainties of some elements in the right-hand-side constraints are expressed as probability distributions, chance-constrained programming (CCP) can be used[18]. A standard CCP constraint Eq. 9 can be converted into a deterministic one, i.e., Eq. 10 when the left-hand-side coefficients [elements of $A_i(t)$] are deterministic and the right-hand-side coefficients [$b_i(t)$] are random (for all p_i values)[19].

$$\Pr\ \{A_i(t)X \leqslant b_i(t)\} \geqslant 1-p_i,\ i = 1,2...,m \tag{9}$$

$$A_i(t)X \leqslant b_i(t)^{(p_i)},\ i = 1,2...,m \tag{10}$$

where $A_i(t) \in A(t)$, $b_i(t) \in B(t)$, $t \in T$; $A(t)$, $B(t)$ are sets with random elements defined on probability space T; $p_i(p_i \in [0,1])$ is a given level of probability for constraint i (i.e., significance level, which represents the admissible risk of constraint violating); m is the number of constraints. $b_i(t)^{(p_i)} = F^{-1}(p_i)$, the cumulative distribution function of $b_i(t)$ [i.e., $F_i(b_i(t))$] and gives the probability of violating constraint $i(p_i)$ [20].

When p_i and $A_i(t)$ are fuzzy with ambiguity and vagueness features, fuzzy chance-constrained programming (FCCP) is generated based on a combination of fuzzy and probability distributions. For transforming the FCCP into FLP, the solution approach can be summarized as: (1) convert the fuzzy number into deterministic number as introduced above; (2) convert the uncertain constraints to deterministic constraint based on the CCP technique.

2.1.5 Fuzzy optimization theory

Fuzzy optimization theory (FOT) is an efficient tool to calculate multi-objective weights[21], which will integrate multiple objectives into a comprehensive coefficient (u_j) quantitatively. Several steps are needed when calculating u_j: (1) calculate relative degree of membership matrix $R = (r_{ij})_{m\times n}$, among which, $r_{ij} = x_{ij}/\max x_j$ corresponds to *bigger means better* indexes, while $r_{ij} = \min x_{ij}/x_j$ corresponds to *smaller means better* indexes; (2) calculate the transposed matrix of $R = (r_{ij})_{m\times n}$, that is, $W = R^T = (\omega_{ji})_{n\times m}$; (3) calculate non-normalized weighting vectors

$$w(i) = \frac{1}{1 + \left[\sum_{j}^{J} (1-\omega_{ji})^p \Big/ \sum_{j=1}^{J} \omega_{ji}^p\right]^{\frac{2}{p}}} \tag{11}$$

where, p is distance parameter, with $p = 1$ means Hamming distance and $p = 2$ means Euclidean distance; (4) unitary processing of non-normalized weighting vectors; (5) calculate maximum relative membership degree of comprehensive benefit

$$u_j = \frac{1}{1 + \left\{\sum_{i=1}^{m} [w_i(r_{ij}-1)]^p \Big/ \sum_{i=1}^{m} (w_i r_{ij})^p\right\}^{\frac{2}{p}}} \tag{12}$$

2.1.6 Linear fractional programming

A linear fractional programming (LFP) model can be expressed as:

$$\begin{cases} \max f(x) = \dfrac{cx + \alpha}{dx + \beta} \\ Ax \leqslant b \\ x \geqslant 0 \end{cases} \tag{13}$$

where, A is $m \times n$ matrix; x and b are column vectors with n and m components; c and d are row vectors with n components, α and β are constants. Duality theory[22] is used to solve LFP. Eq. 13 can be converted to Eq. 14 if (1) $dx + \beta > 0$ with any x; (2) objective function is continuously differentiable; (3) the feasible region is non-empty and bounded.

$$\begin{cases} \min g(y, z) = z \\ A^T y + d^T z \geqslant c^T \\ -b^T y + \beta z = \alpha \\ y \geqslant 0 \end{cases} \tag{14}$$

Equation 14 is an ordinary LP with optimal solution $(\hat{y}, \hat{z})$. $\hat{v} = a^T\hat{y} + d^T\hat{z} - c^T$ and $\hat{v} \geqslant 0$ are introduced as slack variables. $\hat{x}$ is the optimal solution of Eq. 13 and $\hat{u}$ is also a slack variable, thus $a\hat{x} + \hat{u} = b$ and $\hat{u} \geqslant 0$. According to complementary slackness theorem, if $\hat{x}_j\hat{v}_j = 0$ and $\hat{y}_j\hat{u}_j = 0$, Eqs. 13 and 14 share the same optimal solutions.

2.2 Single-objective programming models for crop planting structure optimization

2.2.1 Interval linear programming model

When the parameters of crop planting structure optimization tend to be uncertain, fluctuant or the relevant data-poor, the ILP can be used. For example, the market price for different crops changes with the season, the irrigation quota may vary with different inflow levels, etc. The model can be expressed as:

Model 1:

Objective function

$$\max\ F_1^{\pm} = \sum_{j}^{J} w_j^{\pm} a_j^{\pm} \tag{15a}$$

Water supply constraint

$$m_j^{\pm} a_j^{\pm} \leqslant Q^{\pm} \tag{15b}$$

Crop planting areas constraints

$$\sum_{j=1}^{J} a_j^{\pm} \leqslant A_{\max}^{\pm} \quad \text{and} \quad a_{j,\min}^{\pm} \leqslant a_j^{\pm} \leqslant a_{j,\max}^{\pm} \tag{15c}$$

Non-negativity constraint

$$a_j^{\pm} \geqslant 0 \tag{15d}$$

The meanings of all the symbols in the five models developed in this study are shown in Table 1. The solution approach of Model 1 is based on 2.1.1.

2.2.2 Inexact fuzzy chance-constrained programming model

In most cases, irrigation quotas (m_j) and water availability (Q) are in reality stochastic. However, if the data of these random variables are deficient, they can be expressed as interval or fuzzy numbers to express their randomness. Interval numbers are capable of handling uncertainties with a lower data requirement but may encounter difficulties in tackling highly uncertain parameters. Fuzzy numbers have a good balance between information accuracy and data requirement with flexibility in application. Moreover, in arid and semi-arid regions with water shortage, water resources deficiency has restricted the crop planting structure planning and thus reflected the economic development of agriculture. However, in most cases, the decision makers incline to obtain the maximum planting benefit in an allowed violation probability of available water supply. Thus, under such conditions, the IFCCP model was developed for crop planting structure optimization based on Model 1. The developed model can be described as:

Model 2:

$$\max\ F_2^{\pm} = \sum_{j}^{J} w_j^{\pm} a_j^{\pm} \tag{16a}$$

$$Pos\left\{\sum_{j=1}^{J} \tilde{m}_j a_j^{\pm} \leqslant \tilde{Q}\right\} \geqslant 1 - \tilde{p} \tag{16b}$$

$$\sum_{j=1}^{J} a_j^{\pm} \leqslant A_{\max}^{\pm} \tag{16c}$$

Table 1 Definition of symbols used in the models developed

Parameters and variables	Meaning and instructions
$w_j^{\pm}$ /(CNY·hm^{-2})	Economic benefit of crop j
$a_j^{\pm}$, $a_{j,\min}^{\pm}$, $a_{j,\max}^{\pm}$ /(×10^4 hm^2)	The planning areas (decision variable), the minimum planting area, and the maximum planting area of crop j, respectively
Z_0 /(×10^4 CNY)	The optimal objective function solution of ordinary LP
$Pos\{\cdot\}$	Possibility condition
m_j /(×10^{-4} m^3·hm^{-2})	Irrigation quotas
Q /(×10^4 m^3)	Water availability
p	Probability of violating constraint
A /(×10^4 hm^2)	Crop total planting area
u_j	Maximum relative membership degree of comprehensive benefit
W /(×10^4 CNY)	Total input of planting crops

Note: Assume X is a parameter, then $X^{\pm}$ is an interval parameter with X^- and X^+ representing the lower bound and upper bound of $X^{\pm}$, respectively; $\tilde{X}$ is a fuzzy number with $\underline{X}$, X, $\overline{X}$ representing the lowest possible value, the most credible value, and the highest possible value.

$$a_{j,\min}^{\pm} \leqslant a_j^{\pm} \leqslant a_{j,\max}^{\pm} \tag{16d}$$

$$a_j^{\pm} \geqslant 0 \tag{16e}$$

Based on the solution approaches introduced in Sections 2.1.1, 2.1.3 and 2.1.4. Model 2 can be converted into the following two submodels:

The upper bound of IFCCP

$$\max\ F_2^+ = \sum_j^J w_j^+ a_j^+ \tag{17a}$$

$$[(1-\alpha)\underline{m_j} + \alpha m_j]a_j^+ \leqslant [(1-\alpha)\overline{Q_2} + \alpha Q_2]^{[(1-\alpha)\overline{p}+\alpha p]} \tag{17b}$$

$$\sum_{j=1}^J a_j^+ \leqslant A_{\max}^+ \tag{17c}$$

$$a_{j,\min}^- \leqslant a_j^+ \leqslant a_{j,\max}^+ \tag{17d}$$

$$a_j^+ \geqslant 0 \tag{17e}$$

The lower bound of IFCCP

$$\max\ F_2^- = \sum_j^J w_j^- a_j^- \tag{18a}$$

$$[(1-\alpha)\overline{m_j} + \alpha m_j]a_j^- \leqslant \left[(1-\alpha)\underline{Q_1} + \alpha Q_1\right]^{\left[(1-\alpha)\underline{p}+\alpha p\right]} \tag{18b}$$

$$\sum_{j=1}^J a_j^- \leqslant A_{\max}^- \tag{18c}$$

$$a_{j,\min}^+ \leqslant a_j^- \leqslant a_{j,\max}^- \tag{18d}$$

$$a_j^- \leqslant a_{jopt}^+ \tag{18e}$$

$$a_j^- \geqslant 0 \tag{18f}$$

where m_j and p are triangle fuzzy numbers; Q is trapezoidal fuzzy number, with Q_1 and Q_2 representing the lower and upper bounds of the trapezoidal fuzzy number Q when $\alpha = 1$, respectively; $\underline{m_j}$ and $\overline{m_j}$ are the lower and the upper values of fuzzy number m_j under a certain α level; similar explanations also apply to p and Q.

2.2.3 Inexact fuzzy linear programming model

Apart from the uncertainty of parameters/variables, the fuzziness in the constraints and objective functions also exists because of natural conditions and people's ideology, such that the planting area of a certain crop is usually expressed as "approximately" less than or more than a specific value. In such a case, the IFLP model for crop planting structure optimization can be used. In addition, considering the grayness of some parameters and variables, the ILP model and FLP model were integrated into the IFLP model.

Model 3:

$$\max\ F_3^{\pm} = \sum_j^J w_j^{\pm} a_j^{\pm} \tilde{\geqslant} Z_0 \tag{19a}$$

$$m_j^{\pm} a_j^{\pm} \leqslant Q^{\pm} \tag{19b}$$

$$\sum_{j=1}^J a_j^{\pm} \leqslant A_{\max}^{\pm} \tag{19c}$$

$$a_{j,\min}^{\pm} \tilde{\leqslant} a_j^{\pm} \tilde{\leqslant} a_{j,\max}^{\pm} \tag{19d}$$

$$a_j^{\pm} \geqslant 0 \tag{19e}$$

For the solution of Model 3, the IFLP model has to be converted into the two FLP models first with the upper and lower bounds submodels based on ILP method, then transferring each boundary of the FLP model into ordinary LP models according to the theory introduced in 2.1.2.

2.3 Multi-objective programming models

2.3.1 Fuzzy-optimization-theory-based fuzzy linear multi-objective programming model

The essence of FOT is to calculate the optimal relative weights of different objectives to transform the multi-objective programming problems into single-objective problems. Through coupling the FOT with FLP, the developed model can deal with both fuzziness and multi-objective in crop planting structure optimization. The developed model can be written as:

Model 4:

$$\max\ F_4 = \sum_j^J u_j a_j \tilde{\geqslant} Z_0 \tag{20a}$$

$$\sum_{j=1}^J m_j a_j \leqslant Q \tag{20b}$$

$$\sum_{j=1}^J a_j \leqslant A_{\max} \tag{20c}$$

$$a_{j,\min} \lesssim a_j \lesssim a_{j,\max} \tag{20d}$$

$$a_j \geqslant 0 \tag{20e}$$

To solve this model, the FOT is first adopted to transform the multi-objective model into a single-objective model through maximum relative membership degree of comprehensive benefit. Secondly, changing the fuzzy model to deterministic model based on 2.1.2.

2.3.2 Multi-objective fractional programming model

Sometimes, to avoid the subjective factors when determining index weights of multiple objective functions, the quantification of each objective function is necessary. FLP is an effective way to solve the above problem, especially when the overall efficiency is needed. The developed MFP for crop planting structure optimization in this paper includes two objectives: economic benefit (expressed as total revenue of planting crops) and social benefit (expressed as the minimum irrigation water use). The model can be described as follows:

Model 5:

$$\max F_5 = \frac{\max F_{5(1)}}{\min F_{5(2)}} = \frac{\max \sum_{j=1}^{J} w_j a_j - W}{\min \sum_{j=1}^{J} m_j a_j} \tag{21a}$$

$$\sum_{j=1}^{J} m_j a_j \leqslant Q \tag{21b}$$

$$\sum_{j=1}^{J} a_j \leqslant A_{\max} \tag{21c}$$

$$a_{j,\min} \leqslant a_j \leqslant a_{j,\max} \tag{21d}$$

$$a_j \geqslant 0 \tag{21e}$$

The solution approach is on the basis of 2.1.5.

3 Model application in crop planting structure optimization

The study area is located in Minqin County (103°02′–104°12′ E, 38°05′–39°06′ N), Gansu Province, north-west China. Minqin County is one of the most arid regions in China, with anaverage annual rainfall of 113 mm and average annual evapotranspiration of 2644 mm. The total area of Minqin County is 1.59 km^2, with the oasis accounting for 15%. Illumination in Minqin County is conductive to the growth of crops and more than 90% of the total water consumption is for agricultural irrigation. The main crops include spring wheat, spring maize, oil crops, vegetables, all kinds of melons, etc. Optimizing crop planting structure helps alleviate the serious water resources shortage in Minqin County.

Considering the uncertainties and multiple objectives, the five models developed in this study were applied to the real case study in Minqin County to allocate limited land resources to various crops. Figure 3 demonstrates the process of model development and the technology roadmap for this study. The conditions for each model are

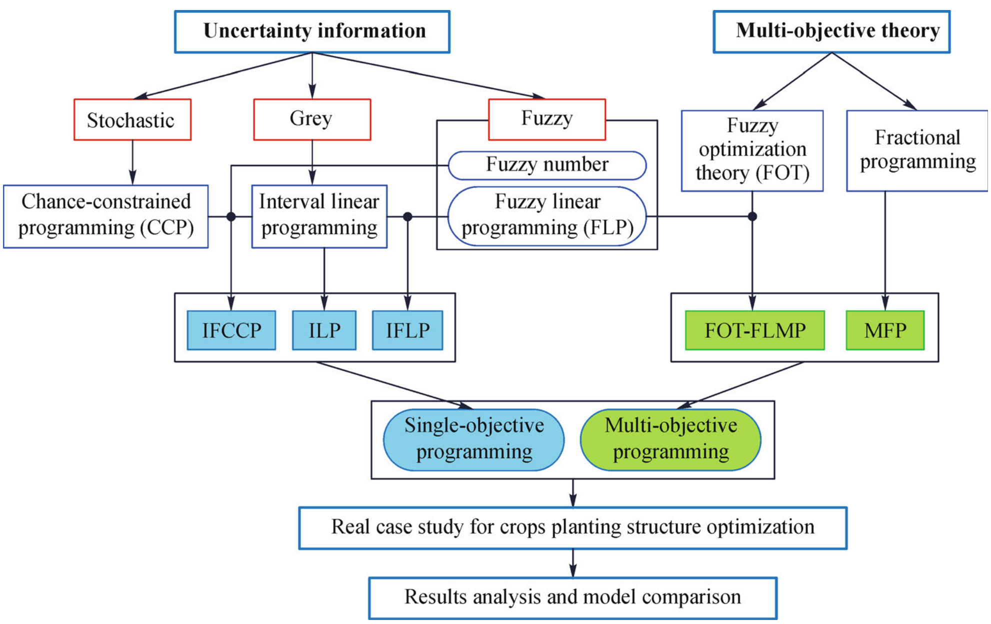

Fig. 3 Technology roadmap

explained in Sections 2.2 and 2.3. In this study, spring wheat, spring maize, oil flax and seed watermelon were chosen as example crops, among which spring wheat and spring maize are grain crops, oil flax an oil crop and seed watermelon is a cash crop. Three flow levels, i.e., low, middle and high flow levels were chosen as different scenarios based on PIII frequency curves of precipitation from 1985 to 2013 as shown in Fig. 4. The probabilities for the low, middle and high flow levels are 75%, 50%, and 25%, respectively. Table 2 and Table 3 give the basic data for calculating the models. In Model 2, $\tilde{Q}$ ($[Q_{1\min}, Q_1, Q_2, Q_{2\max}]$) is trapezoidal fuzzy number; $\tilde{m}_j$ ($[m_{\min}, m, m_{\max}]$) and $\tilde{p}$ ($[p_{\min}, p, p_{\max}]$) are triangular fuzzy numbers.

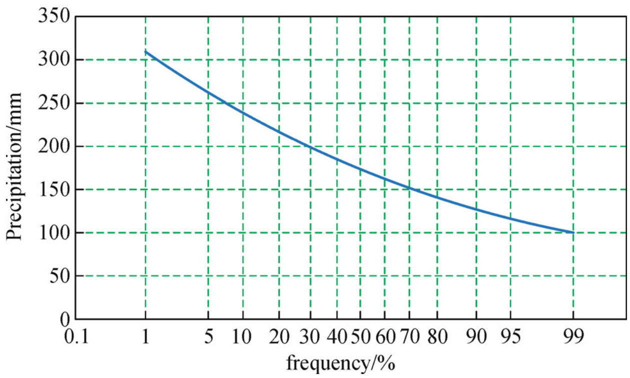

Fig. 4 PIII frequency curve of precipitation

4 Results and discussion

4.1 Solutions of single-objective programming models

The single-objective programming models for optimization of crops planting structure in this study were the ILP, IFCCP and IFLP models. Through interactive algorithm, the results of ILP were be obtained as shown in Table 4. It can be seen that in terms of both optimal crop planting areas and benefits, the corresponding figures decrease from high to middle then low flow levels. This follows the same order as the water availability of these flow levels. The available amount of water for crops at high flow level is the greatest and thus leads to a larger planting area allocation. This also indicates that the water availability directly affects crop planting structure. In addition, all the crops studied, except seed watermelon, change within the corresponding intervals under different flow levels. The results are related with the per unit benefit of each crop. That is, as the per unit benefit of seed watermelon is the highest, the planting area requirement of seed watermelon

Table 2 Basic data for different flow levels

Flow levels	Water supply/($\times 10^4$ m^3)	Water supply (fuzzy)/($\times 10^4$ m^3) ($Q_{1\min}$, Q_1, Q_2, $Q_{2\max}$)	P (fuzzy) ($P_{\min}$, P, $P_{\max}$)
Low	[7082.40, 7252.81]	(6870.17, 7082.40, 7252.81, 7465.04)	
Middle	[7220.72, 7504.90]	(7087.37, 7220.72, 7504.90, 7638.25)	(0.05, 0.10, 0.15)
High	[7368.05, 7571.04]	(7228.03, 7368.05, 7571.04, 7711.06)	

Table 3 Basic data for different crops

Crops	Output value /(CNY·hm^{-2})	The biggest planting area /($\times 10^4$ hm^2)	The smallest planting area /($\times 10^4$ hm^2)	The biggest total planting area /($\times 10^4$ hm^2)	The smallest total planting area /($\times 10^4$ hm^2)	Irrigation water requirement (fuzzy) ($m_{\min}$, m, $m_{\max}$) /(m^3·hm^{-2})
Spring wheat	[48.58, 64.77]	[1.61, 1.65]	1.27			(3500, 3750, 3800)
Spring maize	[63.06, 87.32]	[0.62, 0.63]	0.52	[2.36, 2.39]	[1.95, 2.10]	(3000, 3400, 3563)
Oil flax	[41.10, 61.64]	[0.013, 0.014]	0.007			(2250, 2600, 2850)
Seed watermelon	[71.98, 95.98]	[0.18, 0.20]	0.12			(1350, 1800, 2138)

Table 4 Planting area and system benefit forthe interval linear programming model

Items	Crops	Low flow level	Middle flow level	High flow level
	Spring wheat	1.27	[1.27, 1.33]	[1.31, 1.35]
Planting area ($\times 10^4$ hm^2)	Spring maize	[0.59, 0.63]	[0.62, 0.64]	[0.62, 0.63]
	Oil flax	0.007	[0.013, 0.014]	[0.013, 0.014]
	Seed watermelon	[0.18, 0.20]	[0.18, 0.20]	[0.18, 0.20]
System benefit ($\times 10^6$ CNY)		[276, 316]	[282, 325]	[287, 327]

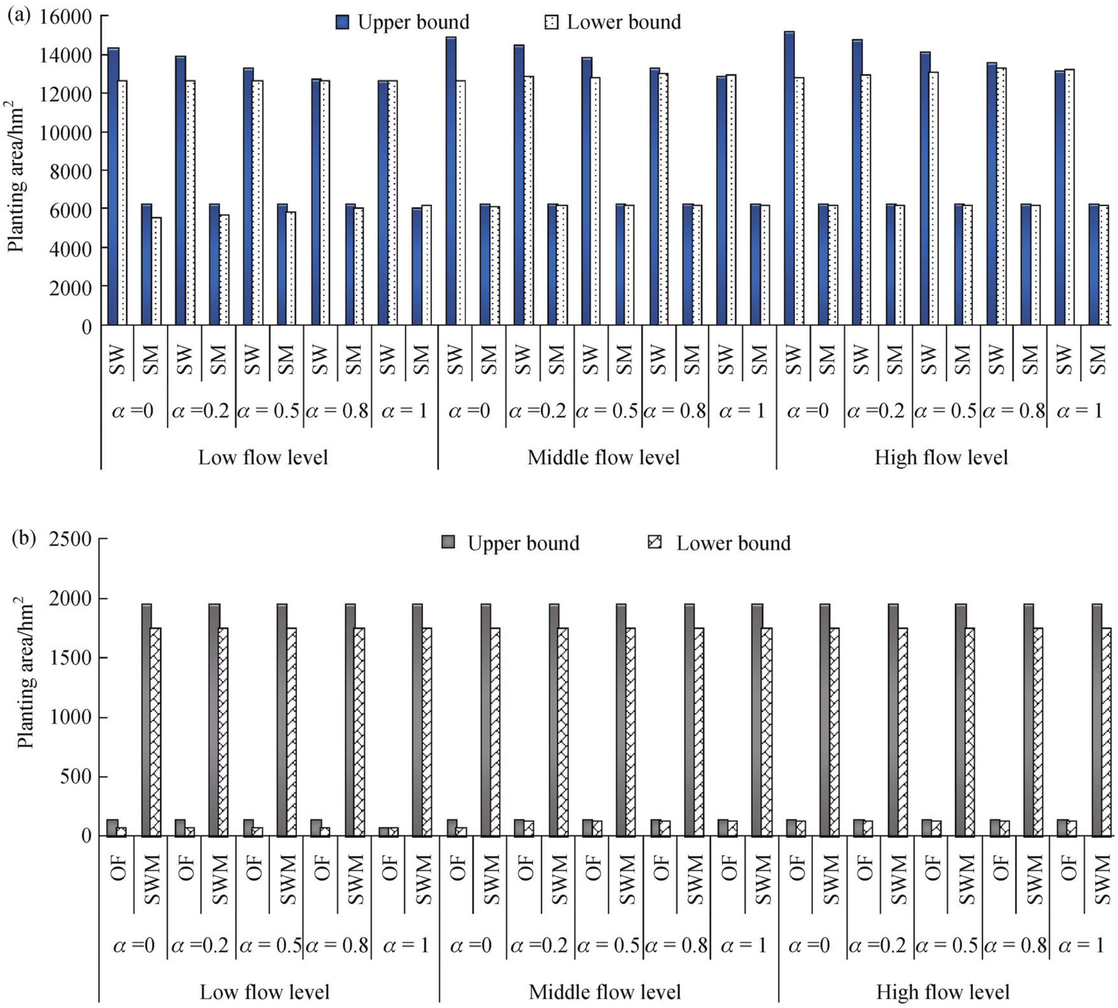

Fig. 5 Planting areas for different crops under differentα-cut values and flow levels of the inexact fuzzy chance-constrained programming model. SW, spring wheat; SM, spring maize; OF, oil flax; SWM, seed watermelon.

should be given priority to obtain the maximum economic benefit under the limited water availability conditions.

Figure 5 shows the results of the IFCCP model, which integrates the ILP and FCCP, and Fig. 6 illustrates the total system benefit under different α-cuts and different flow levels. Five α-cuts values (0, 0.2, 0.5, 0.8, 1) were chosen, which made the information relatively uniform. The upper boundary of both the crop planting area and the system benefit decrease as α-cut value increases, while the corresponding lower boundary increases as α-cut value increases. This is because the larger the α-cut value, the larger the possibility of the occurrence of fuzzy events. Thus, the gap between the upper boundary and lower boundary is narrow when α = 1 and wide when α = 0.

Table 5 shows the results from the IFLP model. In this study, the scaling values for spring maize, oil flax and seed watermelon are set. Given that the IFLP model integrates ILP and FLP, each crop has an upper and a lower boundary scaling value. The upper and corresponding lower

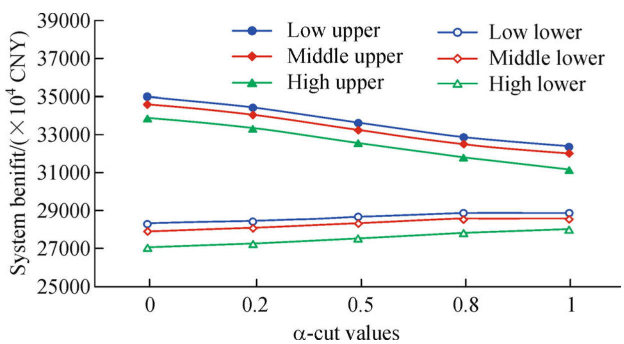

Fig. 6 System benefit of inexact fuzzy chance-constrained programming model. Low upper, the upper boundary of low flow level; Low lower, the lower boundary of low flow level; Middle upper, the upper boundary of middle flow level; Middle lower, the lower boundary of middle flow level; High upper, the upper boundary of high flow level; High lower, the lower boundary of high flow level.

Table 5 Planting area for inexact fuzzy linear programming model

Planting area /($\times 10^4$ hm^2)	Low flow level	Middle flow level	High flow level
Spring wheat	1.27	[1.27, 1.29]	[1.29, 1.31]
Spring maize	[0.58, 0.63]	[0.62, 0.67]	[0.64, 0.67]
Oil flax	0.007	0.007	0.007
Seed watermelon	[0.19, 0.20]	[0.19, 0.20]	[0.19, 0.20]

boundary scaling values for spring maize, seed watermelon and oil flax were set to 380, 113, 7 hm^2, and 133, 47, 6.7 hm^2, respectively. Based on the approach of FLP, d_0 = [48, 109], Z_0 = [276, 316] million CNY at low flow level, d_0 = [137, 281], Z_0 = [282, 325] million CNY at middle flow level, and d_0 = [193, 281], Z_0 = [287, 327] million CNY at high flow level can be obtained. Crop planting area increases as water availability increases except for oil flax, due to its lower per unit benefit.

4.2 Solutions of multi-objective programming models

The FOTB-FLMP model had two objectives in this study: economic benefit (expressed as irrigation benefit contribution values) and social benefit (expressed as agricultural commodities proportion). The agricultural commodities proportion (%) equals the ratio of commodity (t) and production yield (t), as shown in Table 6. In this study, both Hamming distance ($p = 1$) and Euclidean distance ($p = 2$) were calculated and their maximum relative membership degrees were $u_j = (0.5955,\ 0.9859,\ 0.5812,\ 1)$ and $u_j = (0.4354,\ 0.9699,\ 0.4191,\ 1)$ respectively based on the approach of FOT. As the FLP model was introduced, the scaling value of spring wheat, spring maize, oil flax and seed watermelon were set to 27, 380, 7 and 47 hm^2, respectively. Applying the method of FLP, the results of FOTB-FLMP can be seen in Table 7. The maximum membership degrees for $p = 1$ and $p = 2$ are 0.5007 and 0.4995, respectively. It can be seen that crop planting area changes slightly between $p = 1$ and $p = 2$, because of their similar maximum relative membership degree. Thus, decision makers can choose any distance to calculate the model for convenient calculation.

Taking the middle flow level as an example, the MFP model for crop planting structure optimization was calculated. The optimal planting areas of spring wheat, spring maize, oil flax, and seed watermelon are 12667, 5200, 73, and 1240 hm^2, respectively, and the final objective ratio is 4.23 CNY$\cdot$m^{-3}. It can be seen that the results tend to support water-saving. Such results are a positive benefit for arid and semi-arid regions with serious water shortage problems.

4.3 Comparison of the five models

This study developed five models for crop planting structure optimization, with three being single-objective and two multi-objective models. All the three single-objective models are based on ILP, considering the grayness in the system. If the fuzzy uncertainty exists in the parameters, decision makers can choose the IFCCP model, considering the stochastic characteristic of water

Table 6 Economic parameters for different crops

Crops	Economic benefit /(CNY$\cdot$hm^{-2})	Social benefit /%	Commodity /t	Production yield /t	Agricultural commodities proportion /%
Spring wheat	13116	25	24839.35	99359	25
Spring maize	16372	60	28296.00	47160	60
Oil flax	11558	30	63.00	210	30
Seed watermelon	18896	65	57135.00	87900	65

Table 7 Maximum relative membership degree and planting area for different crops for FOTB-FLMP model

Crops	Maximum relative membership degree		Optimal crops planting areas/hm^2	
	$p = 2$	$p = 1$	$p = 2$	$p = 1$
Spring wheat	0.4354	0.5955	13290	13291
Spring maize	0.9699	0.9859	6477	6476
Oil flax	0.4191	0.5813	70	70
Seed watermelon	1	1	1977	1977

Note: FOTB-FLMP model represents fuzzy-optimization-theory-based fuzzy linear multi-objective programming model.

availability as well. If the fuzzy uncertainty exists in the constraints and objective function, the IFLP model can be chosen. Actually, the results from the ILP model area special situation of the IFCCP model, and because the IFCCP model takes all constraints involved in the fuzzy relationships into the optimization process, solutions with the IFCCP model would be more reliable. Using the IFLP model, as there are scaling values of different crops, the final benefit would be larger than the ordinary ILP model. Decision makers should choose the appropriate model according to the particular situation. In this study, the results of the three single-objective models are mostly expressed as intervals which can indicate the uncertainty of the real world more sensitively. A greater area will be allocated to crops to increase the total system benefit and will consequently increase the risk of water shortage, while a smaller area would reduce the water shortage risk. Accordingly, considering different interval results, optimistic decision makers may be inclined to choose the upper boundary of allocated planting area and risk-averse decision makers would choose the lower boundary.

Sometimes, however, single-objective programming models cannot satisfy the demands of sustainable development. To satisfy such demands, multi-objective programming models should be adopted. The attempt to integrate the FOT method with the FLP model gives the FOTB-FLMP model the capacity to reflect both multi-objective and fuzzy uncertainty, although the results tends to be similar to the single-objective programming models because of the planting area limitation of each crop. MFP is another type of multi-objective programming that can express the two objective functions quantitatively, and gives the final ratio of the whole system. The advantage of the MFP model is that it can avoid the subjectivity of some multi-objective models in their weight coefficient. Such models are especially effective when efficiency of a system is important.

5 Conclusions

In this study, five models were developed for crop planting structure optimization, including ILP, IFCCP, IFLP, FOTB-FLMP, and MFP models. The first three models were single-objective models under uncertainty (grayness, randomness and fuzziness), while the last two models were multi-objective models, with FOTB-FLMP model also considering fuzzy uncertainty. Each of the models has their applicable conditions and the results of the models when applied to the real case study have demonstrated their feasibility and applicability. As crop planting structure planning systems are complex, inclusion of only water supply and planting area constraints cannot fully reflect a system’s full characteristics. Although this paper focuses on method application and development, more complicated physical models for crop planting structure optimization are desirable to fully reflect the complexity of crop planting structure optimization systems.

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