

# Risk analysis methods of the water resources system under uncertainty

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**Abstract** The main characteristic of the water resources system (WRS) is its great complexity and uncertainty, which makes it highly desirable to carry out a risk analysis of the WRS. The natural environmental, social economic conditions as well as limitations of human cognitive ability are possible sources of the uncertainties that need to be taken into account in the risk analysis process. In this paper the inherent stochastic uncertainty and cognitive subjective uncertainty of the WRS are discussed first, from both objective and subjective perspectives. Then the quantitative characterization methods of risk analysis are introduced, including three criteria (reliability, resiliency and vulnerability) and five basic optimization models (the expected risk value model, conditional value at risk model, chance-constrained risk model, minimizing probability of risk events model, and the multi-objective optimization model). Finally, this paper focuses on the various methods of risk analysis under uncertainty, which are summarized as random, fuzzy and mixed methods. A more comprehensive risk analysis methodology for the WRS is proposed based on the comparison of the advantages, disadvantages and applicable conditions of these three methods. This paper provides a decision support of risk analysis for researchers, policy makers and stakeholders of the WRS.

**Keywords** water resources system, evaluation criterion, optimization model, risk analysis method, uncertainty

## 1 Introduction

The water resources system (WRS) is the sum total of available water from a variety of sources that can be used by humans within a certain region<sup>[1]</sup>. It comprises many

interactive parts with multi-water resources, multi-regions and multi-users, such as rivers, streams, lakes, ground-water regimes, reservoirs, dams and bifurcations, as well as cities, towns, and water users. At the same time, climate change and human activity could affect the systems at a regional scale and lead to more significant spatial and temporal variations of water resources in the basin and thus the associated environmental and ecological conditions. Therefore, available water supply is one of the crucial issues that have been seriously restricting the global social and economic development and is likely to continue to do so in the future. This will lead to increasingly severe water scarcity<sup>[2]</sup> and increased frequency of incidents of water pollution, drought and flood<sup>[3]</sup>. These problems increase the WRS higher complexity and uncertainty<sup>[4]</sup>. Therefore, humans need to confront the ensuing droughts, floods, water pollution, while they exploit and utilize water resources. Such adverse events inevitably lead to risks, such as economic loss and ecological damage. Naturally the following series of questions are elicited<sup>[5,6]</sup>: What are the risks? What are the source of these risks? What adverse events mentioned above will lead to failure? How likely is this? How severe is it? To solve the above questions, the implications of risk should be first understood.

Risk is the opposite of reliability, that is, the risk can be expressed as one minus the reliability. Risk is always intertwined with uncertainties and hazards<sup>[7]</sup>. Risk can be defined as stochastic characteristics of potential output, while uncertainty is derived from the relative lack of knowledge<sup>[8]</sup>. A distinction between risk, uncertainty and hazard was made by Kaplan and Garrick in 1981<sup>[6]</sup>. Hazard emphasizes the sources of risk while risk often refers to the cause of hazardous things, which is a comprehensive embodiment of uncertainty and loss<sup>[9]</sup>. Risk can be minimized, but it cannot be eliminated. When risk is introduced to the WRS, the objects of risk are the undesirable events (i.e., adverse events) under certain temporal-spatial circumstances, and the extent of the risk is measured mainly by the probability of risk and the

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corresponding loss<sup>[10]</sup>. In the WRS, risk is caused by dynamic features of uncertainty. Thus, evaluating uncertainty is essential for the qualitative description or quantitative expression of risk.

There are always uncertainties in the WRS. What are the uncertainties? What are the sources of these? Much research has attempted to answer these questions. For example, there are two major sources of uncertainty in the WRS: randomness and lack of knowledge<sup>[11]</sup>. The sources of uncertainty in the WRS can be summarized into three aspects<sup>[12]</sup>: natural environmental uncertainty (e.g., precipitation, stream flow, water demand and climate change); social economic uncertainty (e.g., population changes, economic development, policy variations and sudden war); and limitations of human cognitive ability (e.g., uncertainties in the model objectives, constraints and parameters/variables). Therefore, it is difficult to grasp all information of the WRS. Generally, the problem can be simplified appropriately by identifying the key factors that generate risk.

Based on risk identification, expressions of uncertainty can be explored. There is uncertainty in the system because of inherent randomness or imprecision in the modeling of physical phenomena<sup>[13]</sup>. Hence, the uncertainty has been divided into the inherent variability and error estimates in the risk analysis<sup>[14]</sup>. However, from the objective and subjective perspectives, the uncertainty can be divided into stochastic uncertainty and subjective uncertainty in the process of risk analysis<sup>[15]</sup>. Stochastic uncertainty indicates the inherent characteristics of the system itself, and can be expressed in different ways. Subjective uncertainty indicates people's cognitive ability to further describe the characteristics of the system. The subjectivity emphasizes the cognitive behavior when considering the system. In many cases, it is hard to collect completely objective information about the WRS, because this is restricted by the system and human consciousness and so on. Therefore, when there is a lack of data, or only availability of imprecise data, researchers can estimate the overall situation based on limited sample sizes (either measured or simulated) and further evaluate the risk in the WRS<sup>[16]</sup>.

In the past decades, a broad spectrum of literature has been published for risk analysis and risk evaluation of the WRS, considering water shortage<sup>[4,17,18]</sup>, reservoir operational management<sup>[19,20]</sup>, water pollution management<sup>[21–24]</sup> as well as floods<sup>[25–29]</sup>. Some of the basic variables frequently fluctuate with seasonal variation in the WRS, such as stream flow and concentration of pollutants. In addition to analyzing a large amount of data by statistical methods, simulation and prediction of random events can also be performed<sup>[30,31]</sup>. The initial stochastic analysis methods merely focused on digital features like mean and variance, but they were incapable of predicting the likelihood and severity of system failure. System failure can be described by three criteria: reliability, resiliency and vulnerability. These criteria are used to

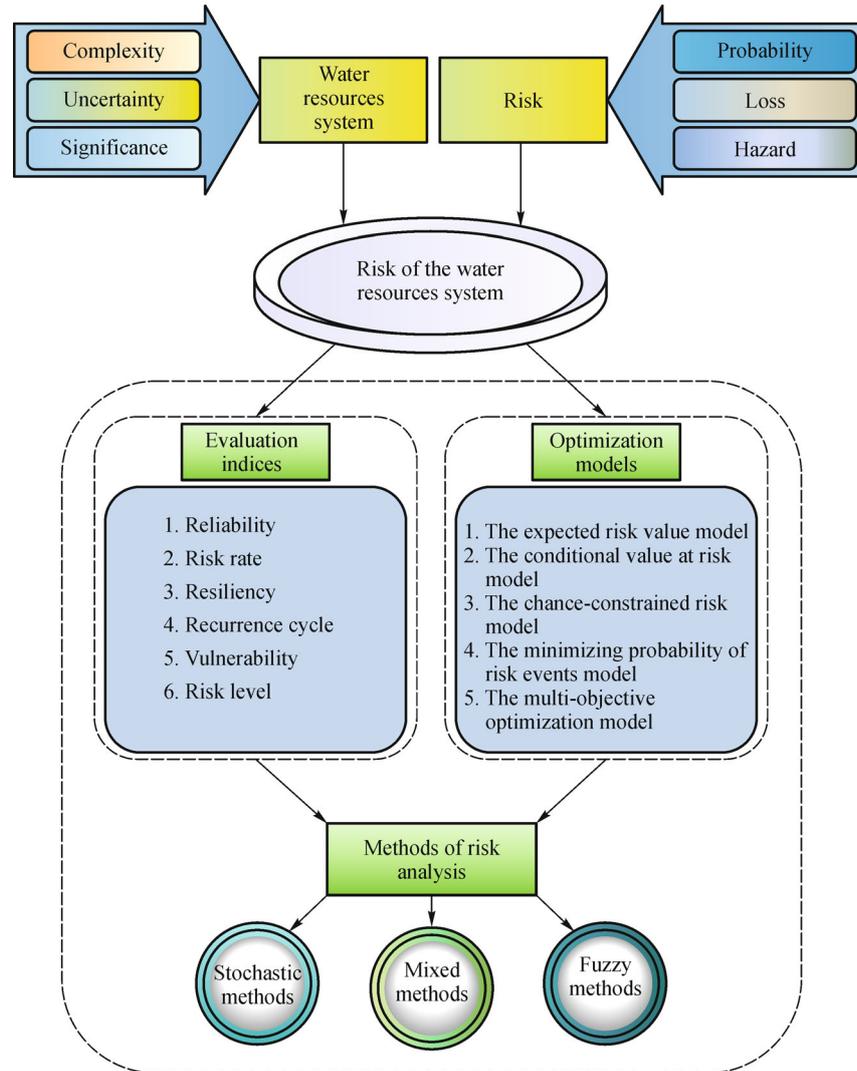
evaluate system performance of a water reservoir<sup>[4]</sup>. If the available data collected are inadequate, other insurmountable problems arise, and the applicability of the stochastic methods will be significantly diminished. Therefore, three fuzzy reliability measures have been developed, including reliability-vulnerability index, robustness index and resiliency index. These indices are capable of handling complicated fuzzy sets and system conditions, and it is an effective way to solve problems for which stochastic methods are inapplicable<sup>[32]</sup>. However, a given fuzzy set implies the need for membership function of the specific expression. Such fuzzy methods that greatly depend on human subjectivity will be brought into question even when expert opinion has been used. Also, the study issues are complex, for example, the characteristics of multi-variables in the risk evaluation model are diverse, and some of these have strong randomness, while others have strong fuzziness. Using only one method (a stochastic method or a fuzzy method) will simplify the problem by ignoring the features of the secondary variables, which will naturally affect the reliability and accuracy of the results. Accordingly, a mixed method combining stochastic methods with fuzzy methods has been developed. For instance, the Monte Carlo stochastic simulation method can be used to help generating fuzzy numbers, and obtain more abundant results and the probability of risk<sup>[33]</sup>. Based on stochastic simulation to estimate the inherent random uncertainty of parameters, human-induced uncertainty by fuzzy analysis is presented. The risk assessment results will be more abundant because of the combination of objective with subjective uncertainties<sup>[26]</sup>. However, the majority of the literature describes risk evaluation models for specific or particular problems. Methods to fully quantify the probability and loss for the risk in the WRS have rarely been reported.

Therefore, considering the relationship between risk and uncertainty (Fig. 1), this paper first introduces the commonly used risk analysis evaluation indices and optimization models and, based on this, the existing methods of risk analysis of the WRS are summarized as stochastic, fuzzy and mixed methods. Then, the applicable conditions of various mathematical analysis methods from quantitative perspective are analyzed in depth. Through comparing the advantages and disadvantages of these three methods, this study provides decision support of risk analysis for the researchers, policy makers and other stockholders by formulating a more comprehensive risk analysis methodology for the WRS.

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## 2 Risk evaluation indices and optimization models

Identification of risk should be considered first when analyzing the risk in the WRS under uncertainty. That is, all possible sources of uncertainty resulting in system



**Fig. 1** The framework of water resources system (WRS) risk analysis

failures should be clearly shown. Taking water shortages as an example to identify risk, Fig. 2 shows the conceptual model for identifying the risk factors underlying the risk of water shortages<sup>[34]</sup>.

Figure 2 illustrates the complexity of risk analysis in the WRS. The procedure can be detailed as sources of risk, risk factors, key risk factors and risk consequences. If the loss is selected to indicate the final risk in the WRS, the relationship between the loss and corresponding frequency will be established. The key factors that affect the loss from water shortages are the reduced availability of water and the increased price of that water. Meanwhile, according to comprehensive analysis of all the risk factors, the key factors can be determined. The sources of risk should be listed, including hydrological, hydraulic, structural and supply-demand factors, prior to identifying the risk factors.

The second step is to quantify the risk and there are various risk evaluation indices and optimization models which can be used to measure the effects of the

uncertainty<sup>[32]</sup>. Decision makers can choose the evaluation indices and optimization models, individually to evaluate system performances, or the evaluation indices can be used as objective functions or constraints in the optimization models framework.

## 2.1 Risk evaluation indices

Many indices can be used to depict the performance of the WRS. Among them, the mean and the variance are frequently used. In general, the mean and the variance describe the system average level and average deviation from the mean of the parameters, respectively, but they can hardly provide sufficient system information about extremes and probability distribution when the risk failure will occur<sup>[4]</sup>.

Three criteria of risk evaluation can be considered for evaluating the possible performance of the WRS as shown in Table 1<sup>[4,10,35–37]</sup>:

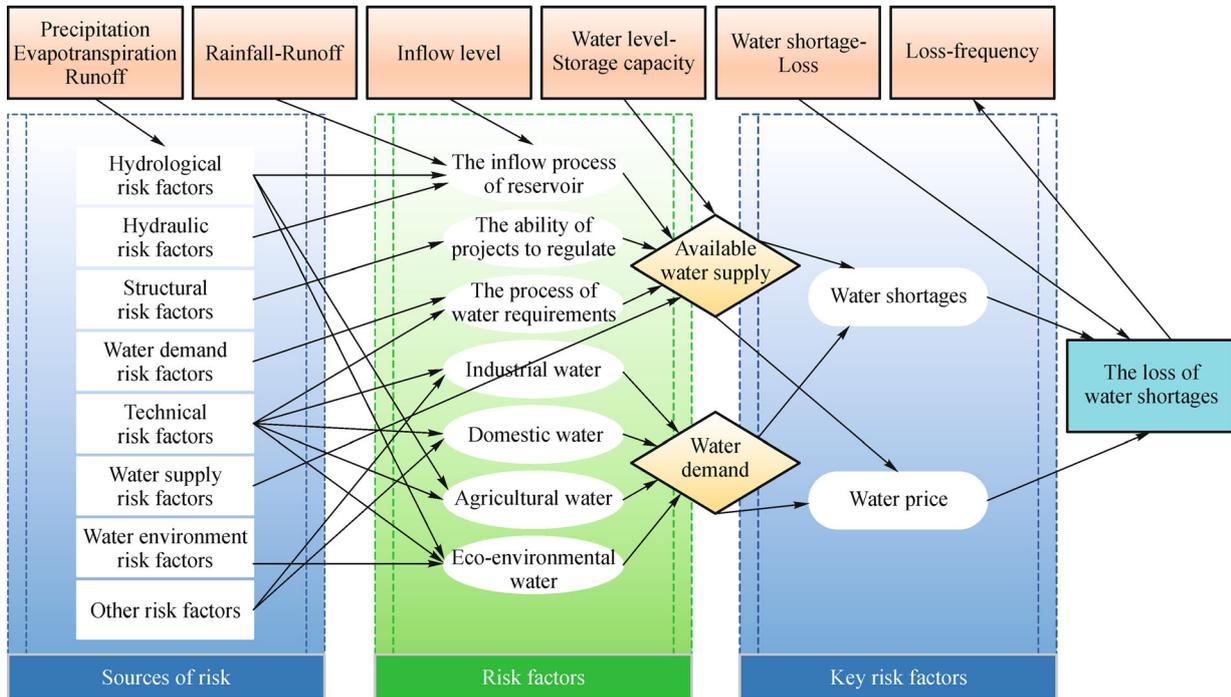


Fig. 2 Conceptual model for identifying the risk factors in the risk of water shortages

Using three aspects (i.e., uncertainty, sensitivity and severity), the risk evaluation indices can be divided into three criteria (reliability, resiliency and vulnerability). With different mathematical expressions for the same criterion, they can be further subdivided into six sub-criteria (reliability, risk rate, resiliency, recurrence cycle, vulnerability and risk level). Taking the reliability as an example, both reliability and risk rate indicate the concept of probability that the WRS is in the normal or an unsatisfactory state, respectively, which satisfies the mathematical equation of  $\alpha + r = 1$ . If  $\alpha = 1$  ( $r = 0$ ), then the system is in the normal state and has high stability. On the contrary, if  $\alpha = 0$  ( $r = 1$ ), then the system is in an unsatisfactory state. In most cases, the values of  $\alpha$  ( $r$ ) are typically between 1 and 0.

Reliability and risk rate are opposites with different focuses on the running state of the WRS. Reliability holds the concept of trust while the risk holds the concept of doubt. However, both reliability and risk rate can only indicate the possibility of system security or failure, without reflecting the nature of risk events inadequately. For example, neither reliability nor risk rate can describe how severe or likely the consequences of a failure may be. Hence, it is necessary to introduce other criteria to describe it. When a failure inevitably occurs in the WRS, indices like resiliency and recurrence cycle can be adopted to measure how quickly the system returns to a running state. Furthermore, means to describe the results of a failure and their severity, the vulnerability and risk level are introduced into evaluation indices<sup>[10,36]</sup>.

In practical applications, a single evaluation index provides insufficient information when the possible sources of uncertainty are complex. Therefore, a single index cannot accurately measure and evaluate the risk (system performance). Appropriate combinations can be chosen to carry on the comprehensive evaluation, fully reflecting the risk of the WRS. Additionally, risk evaluation indices can also be regarded as inputs of the optimization model, using the results of optimization models to reflect their advantages and disadvantages. Thus, several risk analysis models are derived.

## 2.2 Optimization models

Based on the uncertainty theory, there are five ways to formulate a risk analysis model.

(1) For the risk variables or risk losses in the water resource allocation problems, the expected risk value model was introduced<sup>[38–42]</sup>. The objective function can be formulated as minimizing the expected value of risk losses, and the constraints can also be expressed by the expected value.

(2) Safeguarding measures should be taken to reduce the risk in the WRS. In most cases, it is essential for minimizing the expected maximum possibility of risk losses, or minimizing the average value that exceeds a certain risk loss threshold under a certain level of confidence. The conditional value at risk model can be formulated in response to the above issues<sup>[43–45]</sup>.

(3) If it is allowed to violate the constraints under a

**Table 1** The risk evaluation criteria of the WRS

Evaluation criteria (RRV)	Sub-criteria	Formulae	Interrelationship	Features	Applicable conditions
Reliability	Reliability	$\alpha = P(X_t \subset S)$	$\alpha + r = 1$ Reliability and risk rate is relative	If $\alpha = 1$ (or $r = 0$ ), then the system is in a normal state and high stability; otherwise, the system is in an unsatisfactory state	Uncertainty Focusing to describe the probabilities of a failure, but the magnitude and the consequences caused by risk cannot be given
	Risk rate	$r = P(X_t \subset F)$			
Resiliency	Resiliency	$\beta = P(X_t \subset S   X_{t-1} \subset F) = \frac{P(X_{t-1} \subset F, X_t \subset S)}{P(X_{t-1} \subset F)}$	The longer recurrence cycle last, the resiliency is smaller, i.e., after a long recurrence cycle, it is more difficult to back to normal	If $\beta = 1$ (or $\beta = 0$ ), then the system is in a normal state. If $0 < \beta < 1$ , then the system sometimes an unsatisfactory state, but it is possible to return to normal	Sensitivity Now that the system is in an unsatisfactory state, how long does the state return to normal
Vulnerability	Recurrence cycle	$T = \frac{1}{N-1} \sum_{n=1}^{N-1} d(\mu, n)$	Generally it can be described by expected value, standard deviation and coefficient of variation	$0 \leq \chi \leq 1$ , if $\chi = 1$ , than the system has be in a vulnerable state; if $\chi = 0$ , it is always in the normal state	Severity How much the severity and consequences resulted from a failure
	Vulnerability	$\chi = E(S) = \sum_{r=1}^{NF} P_r S_r$			
Risk level	Risk level	$\sigma = \sqrt{D(X)} = \sqrt{\sum_{i=1}^n (X_i - E(X))^2 \cdot P(X_i)}$			
		$C_v = \sigma / E(X) = \sigma / \mu$			

Note:  $\alpha, r, \beta, T, \chi, \sigma, C_v$  = reliability, risk rate, resiliency, recurrence cycle, vulnerability, risk level, respectively;  $X_t$  is the state variable of the WRS at time  $t$ . All  $X_t$  belongs to the set  $X$ . That is,  $X_t \in X$ ;  $F$  and  $S$  denote the state of failure occurs and the normal state of the WRS, respectively;  $d(\mu, n)$  represents the duration of the  $n$ -th interval,  $N = N(\mu)$  is the number of failures belonging to the state  $F$  within the period from 0 to  $t$ ;  $S_i$  and  $P_i$  represent the losses and corresponding probabilities when the  $i$ -th failure occurred;  $E(\dots)$ ,  $D(\dots)$  and  $P(\dots)$  denote the expected value operator, variance operator and probability operator, respectively.

certain level of confidence  $\alpha$  ( $\alpha \in [0, 1]$ ), the probability for constraints should be less than or equal to  $1-\alpha$ . Then, the chance-constrained risk model minimizing value at risk chance can be formulated<sup>[46–49]</sup>.

(4) Decision makers hope that the probability of risk events should be as small as possible, i.e., minimizing the probability of risk events under uncertainty constraints. For this reason, the minimizing probability of risk event model has been developed.

(5) From the perspective of the sustainable use of resources, pursuing the maximum comprehensive benefits, including economic, social and environmental, are more desirable than simply pursuing economic benefit. In such cases, it is necessary to formulate a multi-objective optimization model based on uncertainty systems<sup>[50,51]</sup>. For each single objective, according to the actual situation, decision makers can choose any model or any combination of the four basic models as listed in Table 2<sup>[51,52]</sup>.

Based on the above risk evaluation indices and optimization models, risk analysis methods should be considered after quantifying the risk events.

### 3 Risk analysis/assessment methods of the WRS

The main risk analysis/assessment methods of the WRS include subjective probability method (Delphi method)<sup>[8]</sup>, parameter analysis, extreme value statistics<sup>[53,54]</sup>, uncertainty method, support vector machine<sup>[55]</sup>, and maximum entropy risk analysis method<sup>[56]</sup>. Each method has its own advantages and disadvantages, which should also be analyzed specifically. If the risk evaluation indices are random or fuzzy, uncertainty methods are adopted to solve the problems.

Generally, decision makers can quantify the risk by two major variables coinciding with the characteristics of the information available and the data collected. Taking the reliability as an example, (1) when the collected data or samples are sufficient, it can be assumed that the stochastic uncertainties obey a certain probability distribution (e.g.,

normal distribution, exponential distribution, Poisson distribution, gamma distribution or Pearson type- III distribution), then the risk can be quantified by using probability and statistics methods, and (2) when the samples are insufficient to support the stochastic probability distributions, fuzzy sets theory can be adopted as an effective research tool to assess reliability using the concept of fuzzy probability.

This paper focuses on the randomness and fuzziness characteristics of risk analysis in the WRS. The stochastic methods are applied to the evaluation indices and optimization models with stochastic characteristics, while the fuzzy methods are used for the evaluation indices and optimization models with fuzzy performance. The applications of the stochastic and fuzzy methods as well as mixed methods are discussed below.

#### 3.1 Stochastic methods

The significant characteristic of a stochastic method is to reflect the probability of risk events, quantifying them as the probability distribution or cumulative probability function, which can be obtained by empirical estimation or theoretical simulation. The uncertainty variables can be defined as certain common and typical probability distributions, but it is a great challenge to integrate the probability density function directly. It is difficult to estimate and determine the distribution parameters accurately and less likely to focus on the interrelationship among variables. However, the Monte Carlo stochastic simulation is a common method to solve this problem. Especially, with the rapid development of computer science, it is easy to simulate the actual situation and to reveal the principles of the system by generating a large number of random numbers.

For example, the Monte Carlo stochastic simulation method has been used to generate 10000 sets of risk factors to solve the multi-objective risk analysis model of water resources optimization allocation, and the fundamental target of sustainable utilization of water resources was realized<sup>[57,58]</sup>. The Bayesian principle<sup>[59,60]</sup>, the maximum

**Table 2** The risk optimization models of the WRS

Optimization models	Objective functions	Constraints	References
The expected risk value model	$\min E[f(x;\xi)]$	$E[g_j(x;\xi)] \leq 0$	[38–42]
The conditional value at risk model	$\min \phi_\alpha(x;\xi)$	$g_j(x;\xi) \leq 0$	[43–45]
The chance-constrained risk model	$\min f(x;\xi)$	$\begin{cases} \Pr\{f(x;\xi) \leq f\} \geq \alpha \\ \Pr\{g_j(x;\xi) \leq f\} \geq \beta \end{cases}$	[46–49]
The minimizing probability of risk event model	$\min \Pr\{h_k(x;\xi)\}$	$g_j(x;\xi) \leq 0$	–
The multi-objective optimization model	$\min [f_1(x;\xi), f_2(x;\xi), \dots, f_m(x;\xi)]$	$g_j(x;\xi) \leq 0$	[50–52]

Note:  $E[\cdot]$  denotes the expected value operator; the vector  $x$  represents the decision variables; the vector  $\xi$  is composed by the uncertain risk variables;  $f(x;\xi)$  indicates the objective function of the system losses risk;  $g_j(x;\xi)$  expresses the constrained function of risk,  $j = 1, 2, \dots, p$  represents the number of constraints;  $\phi_\alpha(x;\xi)$  is the loss value of  $\alpha$ -CVaR (conditional value at risk) under confidence level  $\alpha$ ;  $\alpha$  and  $\beta$  are predefined confidence levels, representing the violation probabilities of the objective functions and the constraints, respectively;  $\Pr\{\cdot\}$  denotes the probability of risk event  $\{\cdot\}$  under uncertainty;  $k = 1, 2, \dots, q$  represents the number of risk events  $h_k(x;\xi) \leq 0$ .

entropy principle<sup>[61–63]</sup> and Markov chain<sup>[64,65]</sup> can also be introduced into the study framework to solve the stochastic features of risk analysis in the WRS. The Bayesian method has been used to analyze atmosphere-ocean global circulation models under uncertainty, and further analyze the water demand-consumption risk by Monte Carlo stochastic simulation<sup>[59]</sup>. Since by using the Bayesian method it is difficult to get a post-distribution, the Markov chain Monte Carlo method has obvious continuity advantages in describing the rainfall-runoff relationship<sup>[65]</sup>.

Although the principle of the Monte Carlo method is simple, the simulation results depend on the sample size and number, and the hypothesis of the basic variables is pretty sensitive. Moreover, the calculation workload is heavy and time-consuming. Therefore, another advanced and convenient method can be chosen for indirect estimation to improve the computational efficiency of the traditional Monte Carlo simulation, such as advanced first order second moment<sup>[66,67]</sup>, stratified sampling<sup>[68]</sup>, Latin hypercube sampling<sup>[69]</sup> or an artificial neural network<sup>[70]</sup>.

### 3.2 Fuzzy methods

Risk is relative, without clear attribute, definition or obvious boundaries (e.g., difficult to use deterministic value), which is a typical conception of fuzzy sets. The problem can be described by fuzzy theory, especially when the probability distribution is unknown and the sample size is pretty small.

On the one hand, according to fuzzy mathematics, risk evaluation indices can be divided into several levels, and thus a comprehensive risk evaluation of the WRS is performed. Compared with the single risk index, mentioned above, sub-criteria (Table 2) can determine the risk of the WRS more completely, so as to provide decision-making support for water resources planning and management. The risk level of the evaluation indices can be classified as very-low, low, medium, high and very-high. If the weight of each index is determined by an analytic hierarchy process, the subjective factor of the human may cause deviation of the evaluation results<sup>[9]</sup>. Furthermore, the researchers could use support vector machine<sup>[55]</sup> and entropy theory<sup>[56]</sup> to determine weights on the basis of mathematical theory. By incorporating objective weights with subjective weights, the comprehensive weights can be determined. The results generated by this approach are objective, more reasonable and reliable.

When triangular fuzzy numbers<sup>[71]</sup> and generalized trapezoidal fuzzy numbers<sup>[72]</sup> are taken as the study objects, both of them measure the similarities of fuzzy sets for fuzzy risk analysis. When multiple subcomponents in the fuzzy risk system are expressed as fuzzy linguistic terms (e.g., each risk level corresponding to a fuzzy number), the total risk fuzzy numbers are obtained by the

operation of fuzzy arithmetic. Finally, the similarity between total risk fuzzy numbers and each subcomponent is measured, and the total risk level is obtained.

On the other hand, certain fuzzy variables can be considered as the inputs to an optimization model, and the similar symbols ( $\gtrsim$  and  $\lesssim$ ) appeared in the constraints, which are used to express the fuzzy relationship<sup>[73]</sup>. For example, the interval membership function could be employed to describe the discreteness of different scenarios of inflows and the ambiguity of upper and lower boundaries<sup>[74]</sup>. By incorporating fuzzy numbers with the chance constrained programming, the capacity expansion problems coping with floods can be described<sup>[47]</sup>.

In general, fuzzy sets theory can be applied widely. It is possible to provide an alternative in the case where a random distribution of variable is difficult to obtain. Compared with the stochastic methods, fuzzy methods have relatively prominent subjectivity, which can sometimes be considered as a major flaw.

### 3.3 Mixed methods

Based on the original single uncertainty method, the stochastic, fuzzy (including gray and interval) methods can be combined into a multiple uncertainties framework. In the absence of available statistical data and a proper physical model, fuzzy probability is used to depict and evaluate damages caused by floods, tornadoes, earthquakes and other natural disasters<sup>[75]</sup>. Similarly, the characteristics of stochastic and fuzzy methods could be taken into account, to describe fuzziness and random risk probability distribution of the WRS by the membership function and logistic regression<sup>[18]</sup>. Based on a similar conception, a two-stage integer programming model, infinite two-stage stochastic programming model and interval stochastic fuzzy programming have been proposed for flood management, agricultural water management and water resources carrying capacity risk assessment<sup>[76–78]</sup>.

The predominant advantage of the mixed methods is to accurately reflect the actual characteristics of risk assessment in the WRS. With the support of existing data, the various parameters/variables can be expressed in the form of random variables or discrete interval/fuzzy sets, reflecting multiple uncertainties in the WRS, expressing complex variables inside the system and their relationships. The workload of processing data will increase considerably, however, and the mathematical model used for processing variables and its coupled relationship will be more complicated.

Mathematical analysis methods of risk assessment for the WRS are still developing rapidly, such as robust risk analysis method<sup>[79]</sup>, the chaotic method, genetic algorithm<sup>[80,81]</sup>, wavelet analysis, and the geographic information system.

For practical problems, appropriate analysis methods

**Table 3** Comparison among the three methods of risk analysis in the WRS

Methods	Features	Applicable conditions	An overview of the application	
			Advantages	Disadvantages
Stochastic	1. Sensitivity to hypothesis, depending on sample size and the number of sample	1. The number of samples is relatively sufficient	1. The description of the “probability” of risk is more realistic	1. The gap between the assumption and the practice
	2. High precision, sufficient results	2. Probability distribution can be obtained by an empirical or theoretical estimation	2. Mature development	2. Heavy workload and complex computation
Fuzzy	Describing the probability distribution is unknown and the sample size is pretty small	1. Data are relatively insufficient	1. Subjective evaluation can be involved	1. The membership function construction is inconsistent standard yet
		2. Focus on the magnitude of the risk of relativity	2. Widely used in the risk evaluation indices	2. The principles of selecting the indices are inconsistent
Mixed	Combination of the above two methods	Describing fuzziness and risk random probability distribution	Accurately reflect the actual characteristics of risk assessment in the WRS	1. The workload of processing data will increase worrisomely 2. The mathematical model used for processing variables and its coupling relationship will be more complicated

should be selected based on the specific assessment objectives, available information and data, the study subject and the results of requirements. A brief review of the above three risk analysis methods shows their applicable conditions as well as their advantages and disadvantages (Table 3) and this is intended to provide decision support for researchers and policy makers in the WRS.

#### 4 Conclusions and perspectives

Research on risk analysis methods in the WRS described in this paper can be summarized into three aspects: the inherent stochastic uncertainty and cognitive subjective uncertainty of the WRS from the objective and subjective perspectives are discussed; three criteria including six sub-criteria and five basic optimization models are introduced to quantify the risk; and various methods of risk analysis under uncertainty are summarized including random, fuzzy and mixed methods. By analyzing the conditions for their application, and the advantages and disadvantages of the application of the three mentioned methods, a more comprehensive risk analysis methodology for the WRS is proposed. This paper is intended to provide decision support for researchers, policy makers and stakeholders of the WRS.

However, the risk events have become increasingly complex since the beginning of the 21st century. Therefore it is argued that improvements in the future can be made in two directions.

(1) Risk evaluation is required to be increasingly complex to cope with the temporal and spatial variation of adverse events. Apart from a warmer global climate,

population, economic activity, vegetation cover, land use and rising sea level will be the mainly drivers of change. Reservoir construction, pollution, irrigation and other water utilization will have a strong impact on the security of the WRS. Additionally, transfer of water rights for agriculture and urban and environmental water demand or supply will have a great influence on the risk assessment. This can be further explored from the perspective of a market economy.

(2) Risk analysis methods need to be more sophisticated to cope with the complexity of the WRS. Considering the scope of risk study in the WRS is expanding gradually, more comprehensive evaluation indices, advanced optimization models and uncertainty methods of risk analysis will be further developed. In particular, economic and social implications are often regarded as dominant factors for the existing indices and models, without considering the effect of the ecological environment. Quantification of ecological effects is difficult to achieve, however, with the implementation of regional ecological protection, ecological factors are bound to become a focus for future risk analysis in the WRS. In addition, the risk management issues (e.g., the analysis of feasible choice, balance the costs and risk benefits relationships, and the impact of the final decision) also deserve future discussion.

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